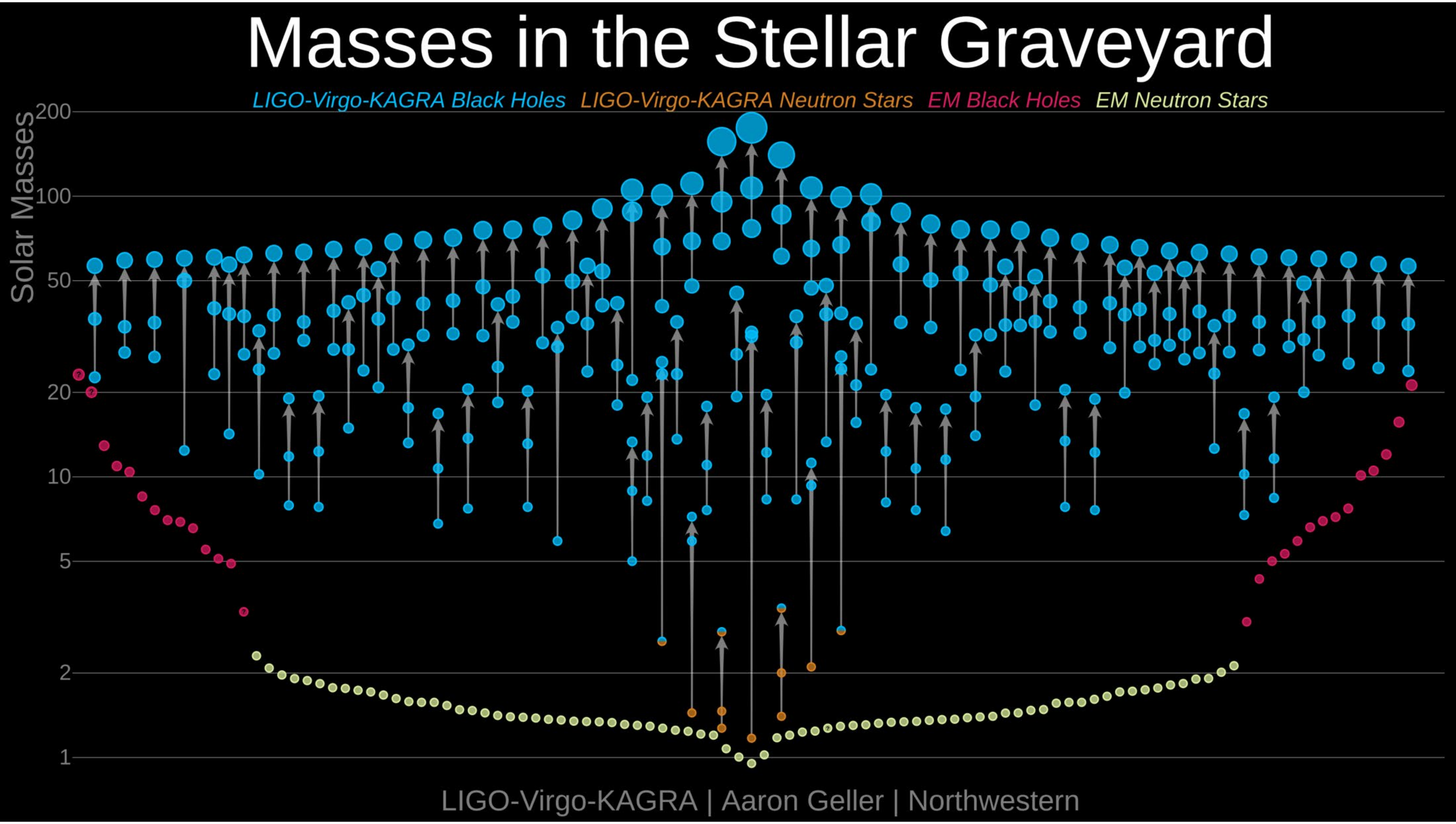
重力波から広がる様々な物理

Takahiro Tanaka
Kyoto University

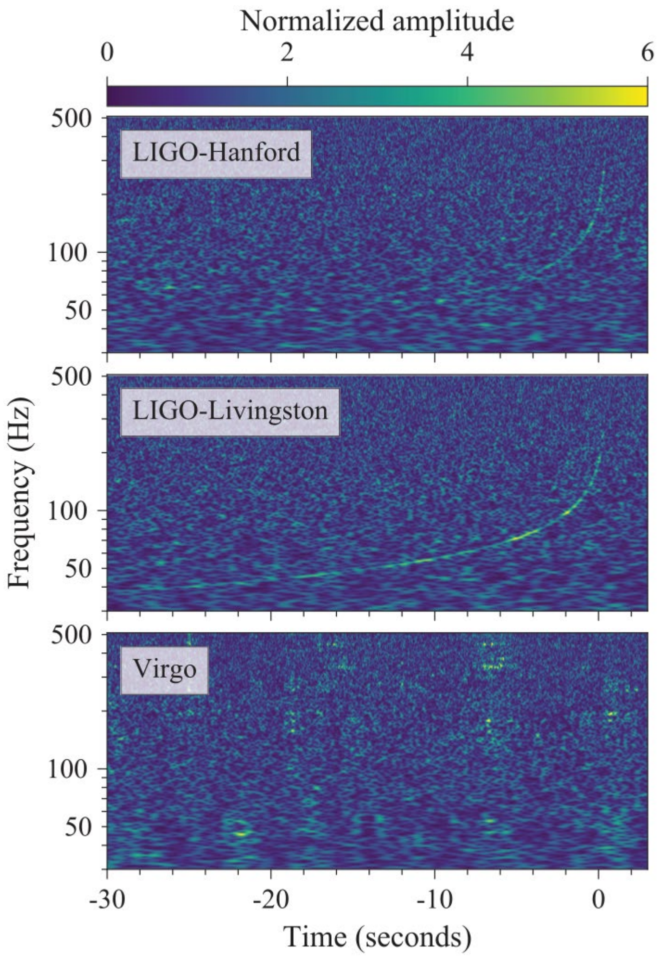


Detected events is getting close to 100.

The next observation run O4 will start soon.

Detection of the first NS-NS binary (GW170817)

Understanding of NS-NS binary increased a lot by this single event.



- Follow-up observations
  - Short γ ray burst 1.7sec after merger
  - Macro(kilo)nova observation by optical and infrared observations
    - *r*-process nucleosynthesis
    - Early blue component
  - Superluminal motion detected by radio observation
    - Existence of a relativistic jet
- Constraint on the nuclear EOS
- Propagation speed of GW $\simeq c$
- Bulk of dark energy model was excluded.
- Possible independent determination of H_0.

Also, various theoretical progresses

Many new models of gravity
$\Rightarrow$ various tests of GR
Binary formation scenarios
Binary coalescence
- Dynamics including various physics
- Emission process, Nucleosynthesis
- Nuclear physics

Supernova explosion
$\Rightarrow$Exploding numerical models
- understanding of various mechanism of GWs

New research areas are opening up.

- Fundamental Physics
- HE astrophysics
- Binary formation

Propagation speed of gravitational waves

1.7sec delay of gamma ray detection over 40Mpc implies

$$\left|c_{GW}^2 - c_\gamma^2\right| < 10^{-15}$$

Extension of gravity theory is tightly constrained.

Ex.) Horndeski theory: scalar-tensor theory with a single scalar field with only upto 2nd order time derivatives in EoM.

$$\mathcal{L}_2 = K(\phi, X),$$
$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$
$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right],$$
$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

$$c_{gw}^2 = \frac{G_4 - X\left(\ddot{\phi}G_{5,X} + G_{5,\phi}\right)}{G_4 - 2XG_{4,X} - X\left(H\dot{\phi}G_{5,X} - G_{5,\phi}\right)}$$

$$\Rightarrow \quad G_4 = G_4(\phi),\ G_5 = 0$$

Only rather traditional models survive.

K-essence, DGP braneworld, scalar-tensor theory

Motivation to consider gravity theories beyond GR

1) Incompleteness of General relativity

GR is non-renormalizabile
Singularity formation after gravitational collapse

2) Dark energy problem

3) To test General relativity

GR has been repeatedly tested since its first proposal.
The precision of the test is getting higher and higher.

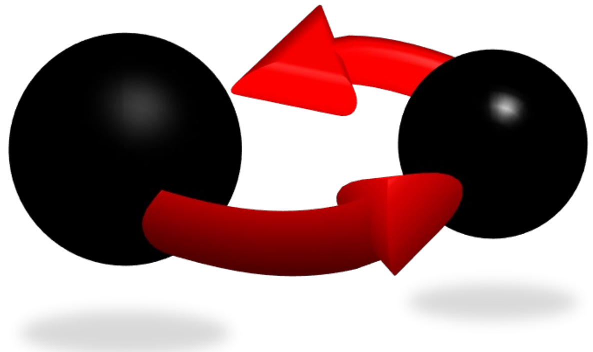
⇒ Do we need to understand what kind of modification is theoretically possible before experimental test?

Yes, especially in the era of gravitational wave observation!

Test of GR using coalescing binaries

- Inspirals of binary BHs and NSs (Cutler et al, PRL 70 2984(1993))

  Clean system: ∼point particles



Waveform in Fourier space for quasi-circular inspiral

$$\boxed{h(f) \approx A f^{-7/6} e^{i\Psi(f)}} \qquad \boxed{A = \frac{1}{\sqrt{20\pi^3}} \frac{\mathcal{M}^{5/6}}{D_L}, \quad \mathcal{M} = \mu^{3/5} M^{2/5}, \quad \eta = \frac{\mu}{M}}$$

$$\Psi = 2\pi f t_c - \varphi_c + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3} \left[1 + \underbrace{\frac{20}{9}\left(\frac{743}{331} + \frac{11}{4}\eta\right)}_{\text{1PN}} v^2 - \underbrace{(16\pi - \beta) v^3}_{\text{1.5PN}} + \cdots \right]$$

$$v^3 \equiv \pi M f$$

Typical modification of GR

often discussed in the context of test by GWs

Scalar-tensor gravity

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R-\omega_{BD}\phi^{-1}\phi_{,\alpha}\phi^{,\alpha}\right)-\sum_a\int d\tau_a m_a(\phi)$$

$$G=\frac{4+2\omega_{BD}}{\phi(3+2\omega_{BD})}$$

scalar charge of self-gravitating body:

$$s_a=-\left[\partial(\ln m_a)/\partial(\ln G)\right]_0$$

G-dependence of the gravitational binding energy

$$\Psi=\cdots+\frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\left[\alpha u^{-2/3}+1+\left(\frac{3715}{756}+\frac{55}{9}\eta\right)u^{2/3}-(16\pi-\beta)u+\cdots\right]$$

Dipole radiation＝－1 PN frequency dependence

$$u=\pi M f=O(v^3)$$

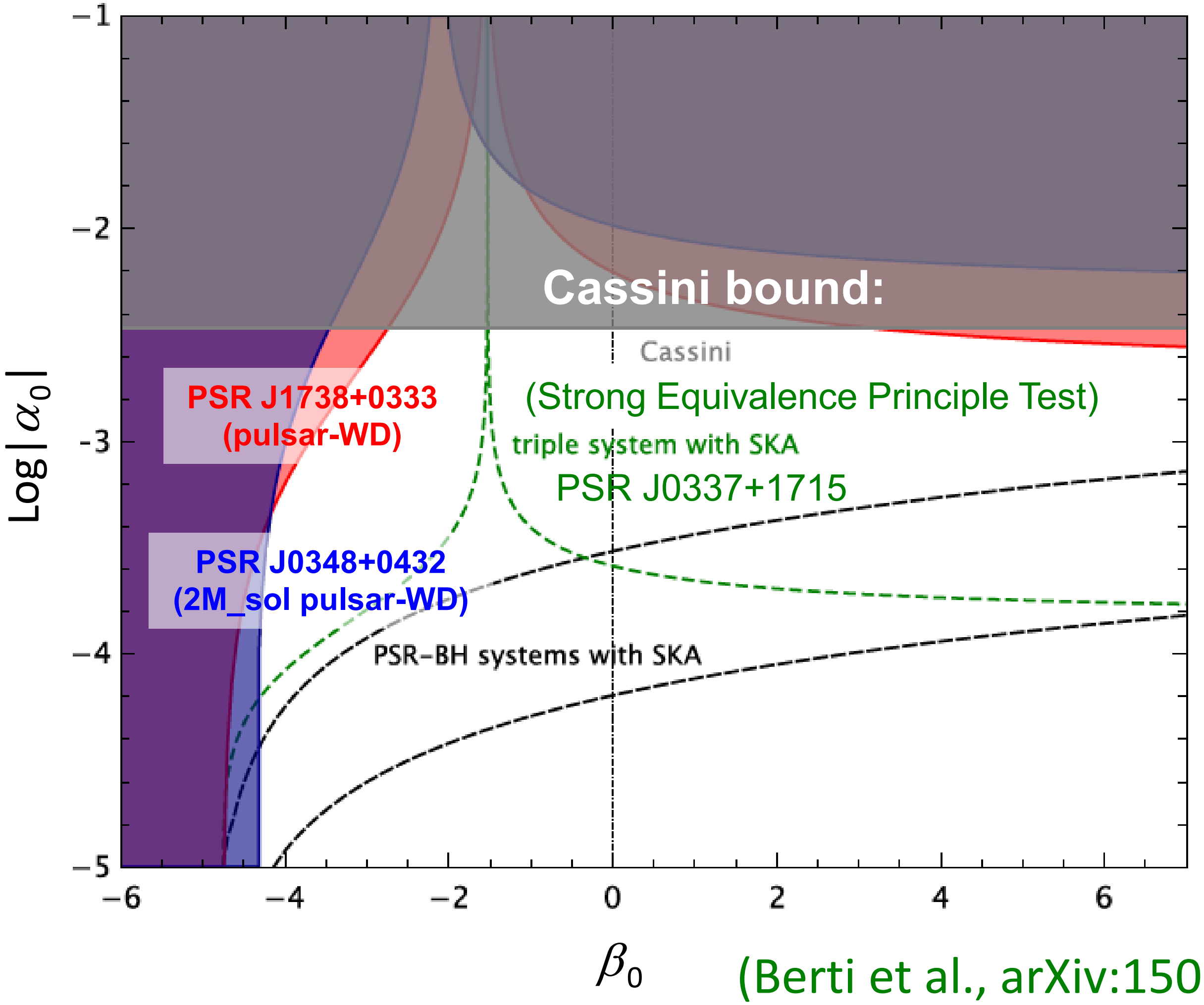
$$\boxed{\alpha=-\frac{5(s_1-s_2)^2}{64\omega_{BD}}}$$

For binaries composed of similar NSs, $(s_1-s_2)^2\ll 1$

After conformal transformation, the action can be recast into the following form:

$$\tilde{I} = (16\pi G)^{-1} \int [\tilde{R} - 2\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)](-\tilde{g})^{1/2} d^4x + I_{\rm m}(\psi_{\rm m}, A^2(\varphi)\tilde{g}_{\mu\nu}),$$

$$\ln A(\varphi) = \alpha_0(\varphi - \varphi_0) + \frac{1}{2}\beta_0(\varphi - \varphi_0)^2 + \cdots$$



$$\alpha_0^2 = \frac{1}{2\omega_{\rm BD} + 3}$$

Constraint from future GW observations:

(Yagi & TT, arXiv:0908.3283)

LISA- $1.4M_\odot$NS+$1000M_\odot$BH:
at 40Mpc
corresponding to $SNR = \sqrt{200}$

$\log|\alpha_0| > -2$

DECIGO-$1.4M_\odot$NS+$10M_\odot$BH:
collecting 10^4events
at cosmological distances

$\log|\alpha_0| > -4.1$

(Berti et al., arXiv:1501.07274)

Spontaneous scalarization

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R-\frac{\omega_{BD}(\phi)}{\phi}\phi_{,\alpha}\phi^{,\alpha}\right)$$

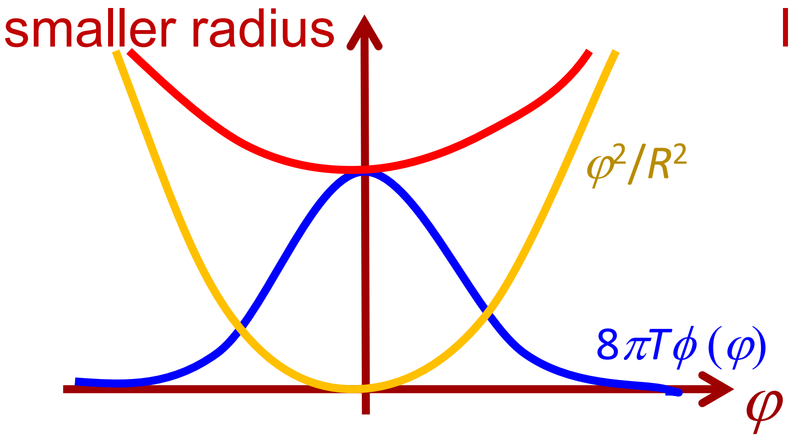
More general model

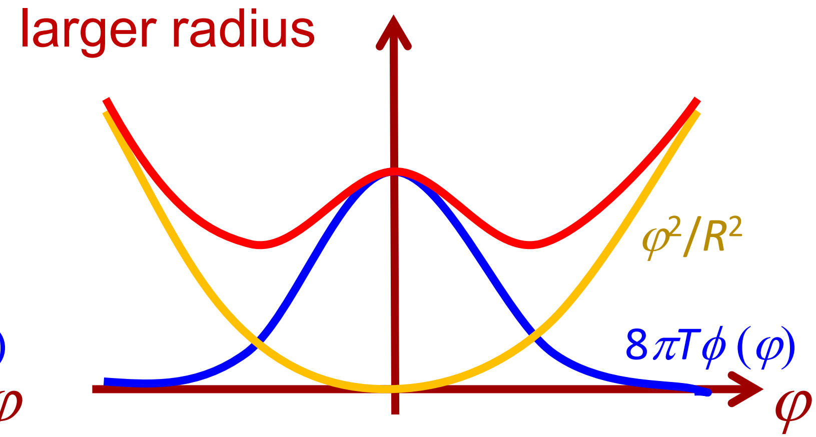
$$=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi(\varphi)R-\frac{1}{2}\varphi_{,\alpha}\varphi^{,\alpha}\right)$$

φ is canonically normalized

EOM $\Delta\varphi\approx-8\pi\phi'(\varphi)T$

Effective potential for a star with radius *R*.





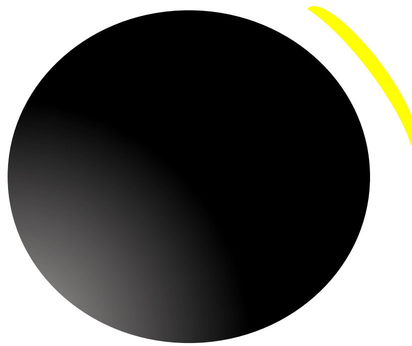
As two NS get closer, “spontaneous scalarization” may happen.
Sudden change of structure and starting scalar wave emission.

Most of parameter region will be excluded by the discovery of many pulsars.

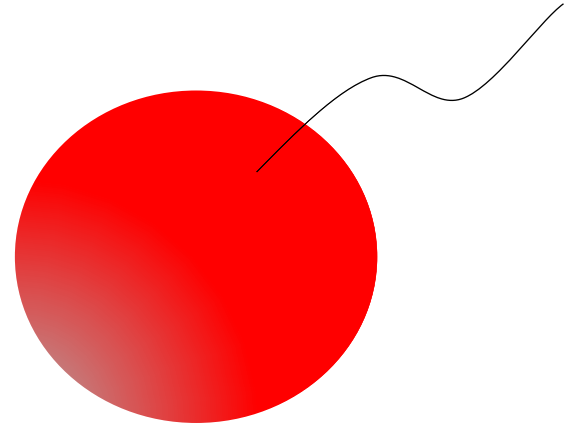
Consistency with the cosmological evolution is difficult to achieve (1607.08888)

Ordinary scalar-tensor theory

BH no hair



Turu-turu



NS can have a scalar hair

Einstein dilaton Gauss-Bonnet, Chern-Simons gravity

$$S \supset \frac{\alpha}{G_N} \underbrace{\int d^4x \sqrt{-g}\, \theta \begin{pmatrix} R_{GB} \\ {}^*R\,R \end{pmatrix}}_{\theta \times (\text{higher curvature})} - \frac{1}{2G_N} \int d^4x \sqrt{-g} \left[(\partial\theta)^2 + 2V(\theta) \right]$$

$$R_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R^{\alpha\beta}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta} \qquad {}^*R\,R = \varepsilon^{\alpha\beta}{}_{\sigma\chi}R^{\sigma\chi}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}$$

- For constant θ, these higher curvature terms are topological invariant. Hence, no effect on EOM.
- Higher derivative becomes effective only in strong field.

Hairy BH - bold NS

- NS in EDGB and CS do not have scalar monopole charge.

$$\Box\theta \approx "R^2" \Rightarrow Q = \int d^3x\, "R^2" = \frac{1}{T}\int \underline{d^4x\, "R^2"}$$

topological invariant, which vanishes on topologically trivial spacetime.

- By contrast, BH solutions in EDGB and CS have scalar monopole and dipole, respectively.

EDGB : monopole charge $\Rightarrow$ dipole radiation (-1PN order)

CS : dipole charge $\Rightarrow$ 2PN order corrections

(Yagi, Stein, Yunes, Tanaka (2012))

Observational bounds

- EDGB

Cassini $\alpha_{EDGB}^{1/2} < 1.3 \times 10^{12}\,\mathrm{cm}$ (Amendola, Charmousis, Davis (2007))

Low mass X-ray binary, A0620-00

$$\alpha_{EDGB}^{1/2} < 1.9 \times 10^{5}\,\mathrm{cm}$$ (Yagi, arXiv:1204.4524)

Ground-based GW observation
GW200115+

$$\alpha_{EDGB}^{1/2} < 1 \times 10^{5}\,\mathrm{cm}$$ (arXiv:2104.11189, 2201.02543, 2302.10112)

- CS

Gravity Probe B, LAGEOS (Ali-Haimound, Chen (2011))

$$\alpha_{CS}^{1/2} < 10^{13}\,\mathrm{cm}$$

Ground GW observation with favorable spin alignment:
100Mpc, $a \sim 0.8M$

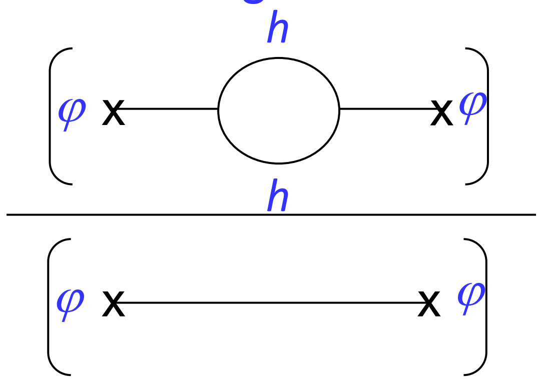
$$\alpha_{CS}^{1/2} < 10^{6-7}\,\mathrm{cm}$$ (Yagi, Yunes, TT, arXiv:1208.5102)

GW+NICER: $\alpha_{CS}^{1/2} < 8.5 \times 10^{5}\,\mathrm{cm}$ (Silva et al., arXiv:2004.01253)

Issue of strong coupling

- Simplified model

Flat background $$S \approx M_{pl}^2\left[(\nabla\varphi)^2+(\nabla h)^2+\alpha\varphi\left(\nabla^2 h\right)^2\right]$$

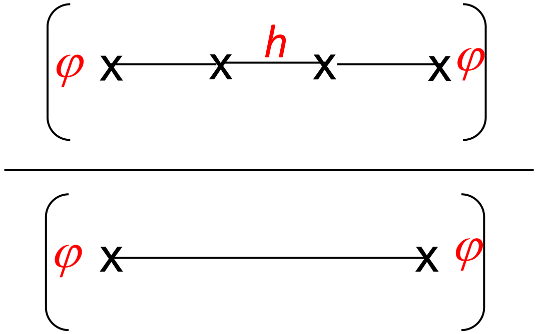


$$\approx\left(M_{pl}^2\alpha p^4\right)^2\times\left(\frac{1}{M_{pl}^2 p^2}\right)^3\times p^4=\frac{\alpha^2 p^6}{M_{pl}^2}$$

strong coupling scale $\approx\left(\dfrac{M_{pl}}{\alpha}\right)^{1/3}$

With background curvature

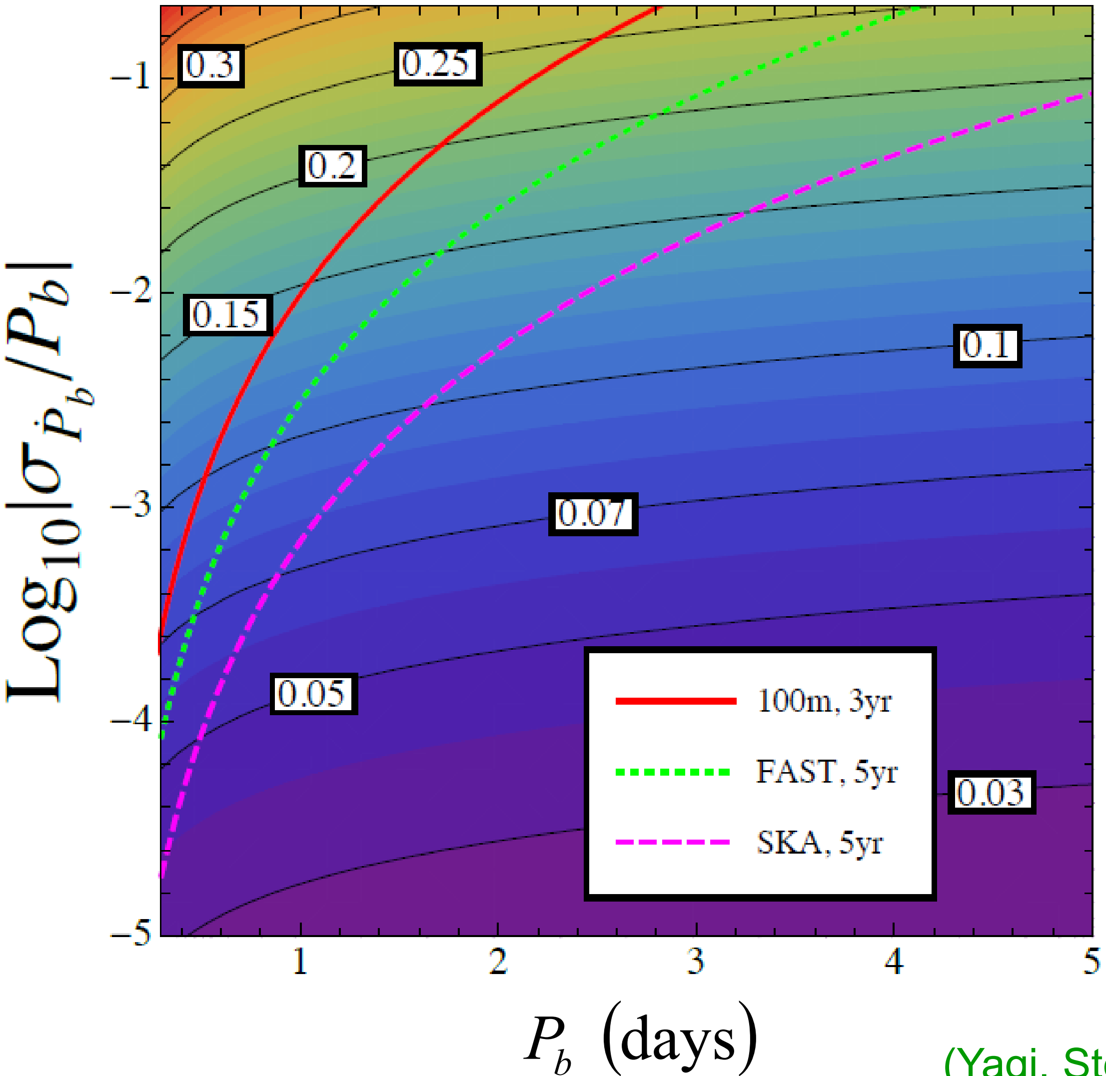
$$S \approx M_{pl}^2\left[(\nabla\varphi)^2+(\nabla h)^2+\alpha\varphi\left(\nabla^2 h\right)R\right]$$



$$\approx\left(M_{pl}^2\alpha p^2 R\right)^2\times\left(\frac{1}{M_{pl}^2 p^2}\right)^2=\alpha^2 R^2$$

strong coupling when $\alpha|R|\geq 1$

Future bounds on EDGB from BH-pulsar system



$\alpha_{\mathrm{EDGB}}^{1/2}$ in the unit of km

Once a BH-pulsar system is found, how precisely one can measure the orbital decay rate determines the strength of the constraint on α_{EDGB}.

(Yagi, Stein, Yunes, arXiv:1510.02152)

Parametrized test of gravitational wave form

$$\boxed{h(f) \approx A f^{-7/6} e^{i\Psi(f)}}$$

GW waveform from a binary in a quasi-circular orbit

$$\Rightarrow \begin{cases} A(f) \to \left(1 + \sum_i \alpha_i u^{a_i}\right) A_{GR}(f) \\ \Psi(f) \to \Psi_{GR}(f) + \sum_i \beta_i u^{b_i} \end{cases}$$

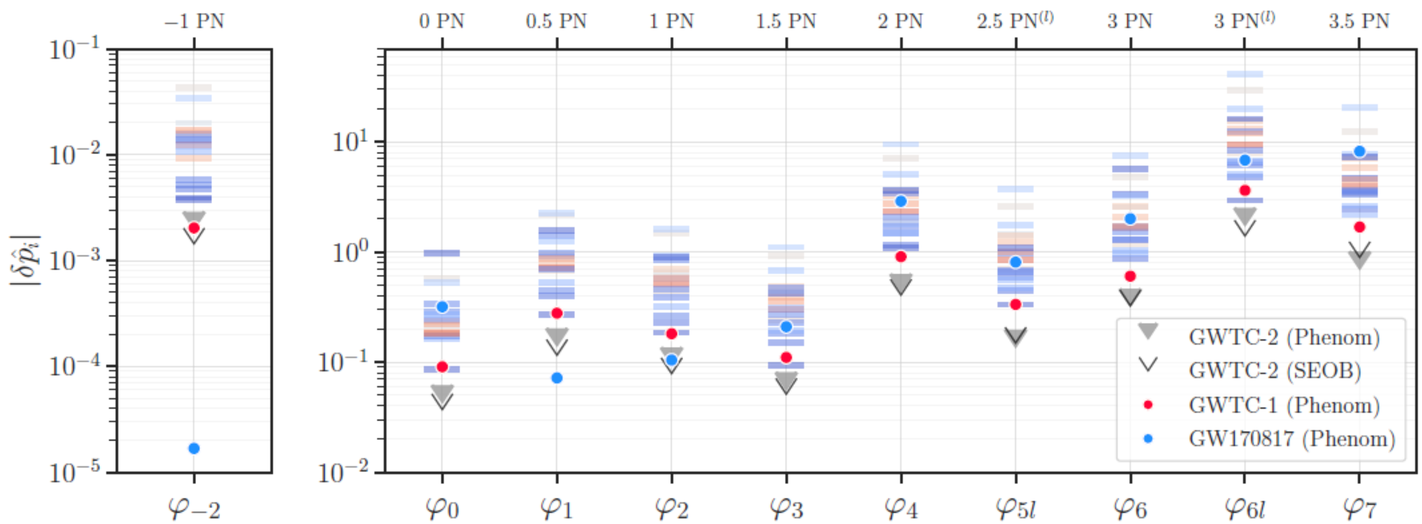
Theory	a	α	b	β
Brans-Dicke [9, 10, 14–16]	–	0	-7/3	β
Parity-Violation [22, 34–37]	1	α	0	–
Variable $G(t)$ [38]	-8/3	α	-13/3	β
Massive Graviton [8–14]	–	0	-1	β
Quadratic Curvature [23, 44]	–	0	-1/3	β
Extra Dimensions [45]	–	0	-13/3	β
Dynamical Chern-Simons [46]	+3	α	+4/3	β

(Yunes & Pretorius (2009))

Calculable leading order corrections are described by the parameter b and β in many cases.

Parametrized Post Einstein (PPE) formalism

Constraint on the relative amplitude of deviation from GR for each PN coefficient of the phase evolution



-1PN 0PN 1PN 2PN

(LIGO/Virgo 2010.14529)

Constraints on the modification to gravitational waves during the inspiral phase.

But these are not the whole story.
There are also many examples of possible extension from GR that go beyond the scope of PPE formalism.

1) Identifying theoretically well-motivated models of extended gravity
2) Theoretical prediction on the modification in the GW waveform
3) If the prediction is beyond the scope of the generic test of gravity, we need to implement the analysis code and applied.

Challenges on testing gravity

Constructing gravitational wave templates in extended gravity scenarios

- Higher dimensional operator corrections (EDGB, CS).
- Low energy extra degrees of freedom that evade all the weak gravity tests (Screening mechanism, Parity odd, Very weakly interacting with matter).
- Modified nature of black hole horizon (ECO).

Massive scalar dipole radiation

Scalar-tensor theory is already strongly constrained by the pulsar timing observations.

Constraint on dipole radiation from GW170817 (1811.00364)

$$B \leq 1.2 \times 10^{-5}$$

is to be compared with pulsar constraint

$$|B| \lesssim 6 \times 10^{-8}$$

However, the scalar field may have a small mass.

The scalar field should be stabilized for $T_{\mathrm{univ}} < 10\mathrm{MeV}$, since the effective gravitational coupling G_N must be unchanged during BBN.

$$\Rightarrow \quad m_{\mathrm{scalar}} > 10^{-16}\mathrm{eV},$$

which is marginally sufficient to evade the constraint from pulsar timing (PSR J0348+0432).

LIGO-Virgo-KAGRA band corresponds to the mass range

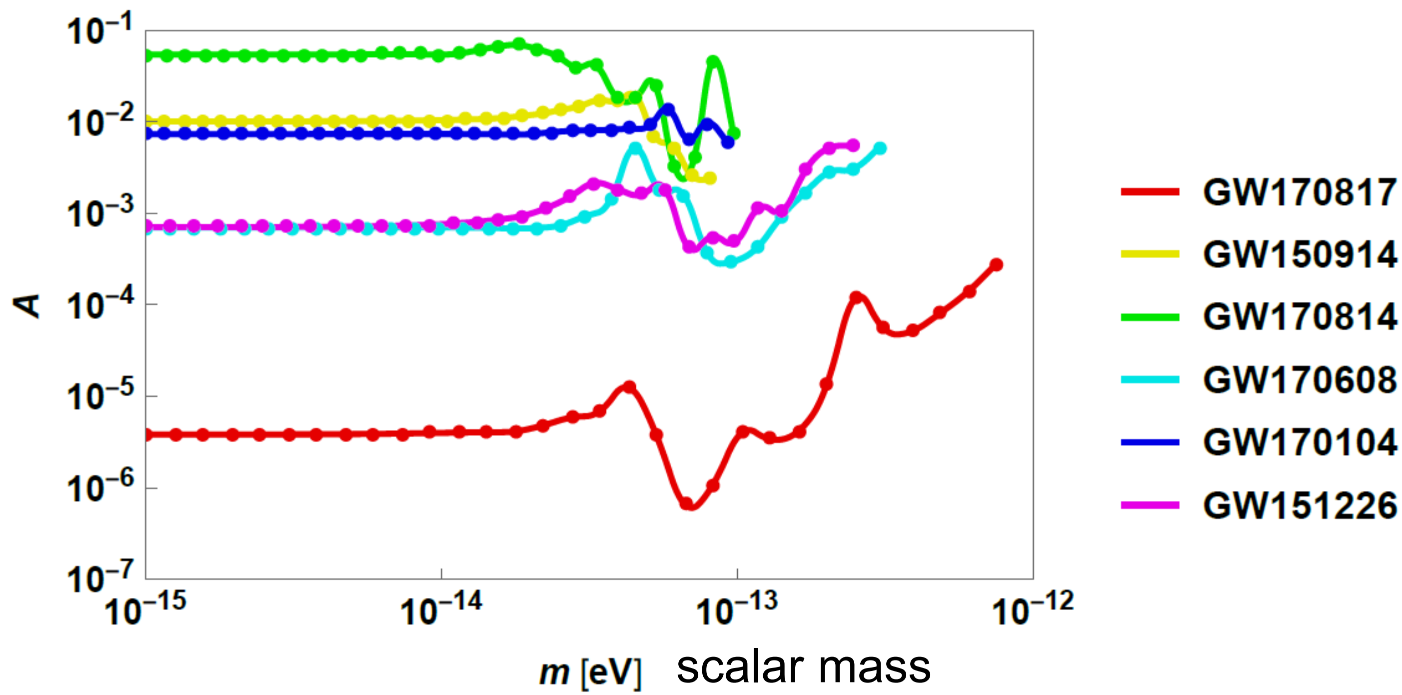
$$10^{-13}\mathrm{eV} > m_{\mathrm{scalar}} > 10^{-14}\mathrm{eV}$$ (Ramazanoglu et al. 1601.07475)

Result of application of the test to O1 and O2 data

K. Yamada, T. Narikawa and TT arXiv:1905.11859

$$\left(\frac{dE}{dt}\right)_d = A\left(\frac{dE}{dt}\right)_Q \frac{k^3}{\omega^3}\Theta(\omega^2 - m^2)(2\pi f M)^{-2/3}$$

90% confidence level upper bound

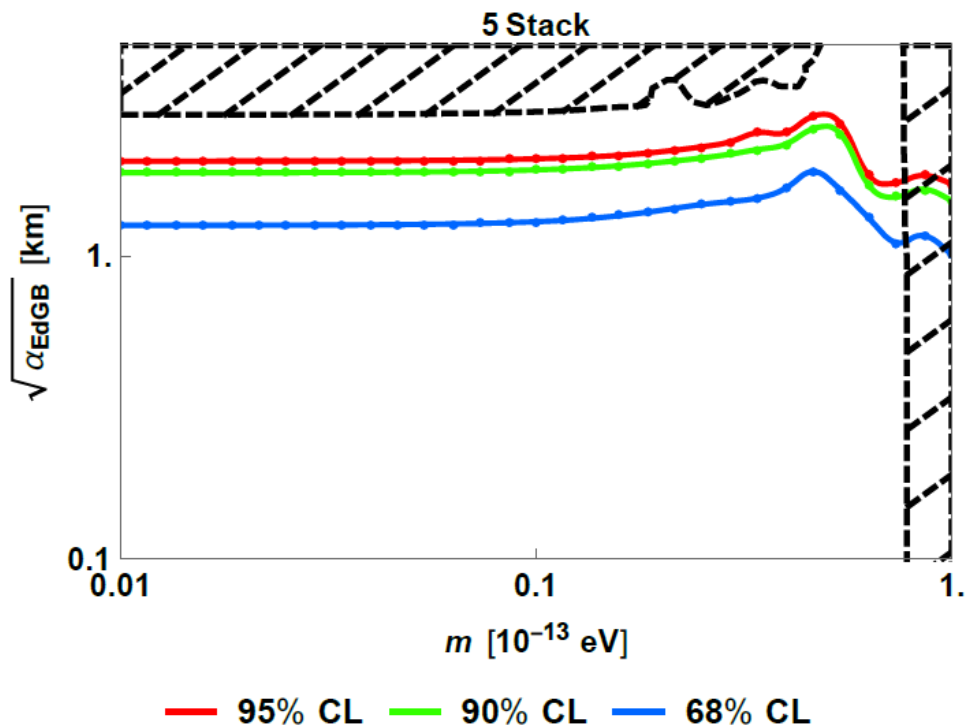


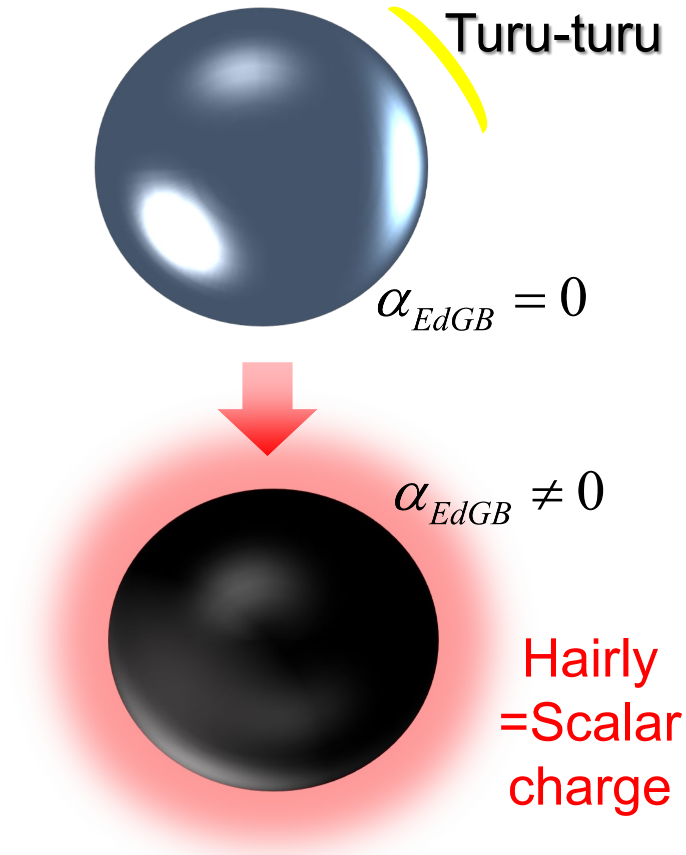
scalar mass

Even we introduce the scalar mass, GR is more preferred.

Einstein dilaton Gauss-Bonnet

$$S \supset \frac{\alpha_{EdGB}}{G_N}\int d^4x\sqrt{-g}\,\theta\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R^{\alpha\beta}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}\right) - \frac{1}{2G_N}\int d^4x\sqrt{-g}\left[(\partial\theta)^2 - m^2\theta^2\right]$$



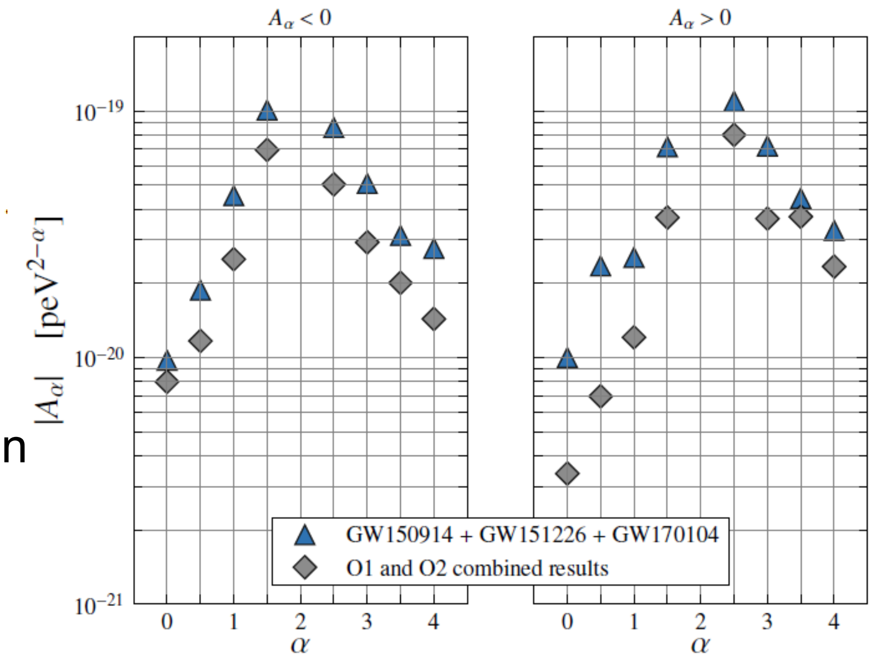


Gauss-Bonnet coupling with massive scalar field in this mass range was constrained for the first time.

Parity violating dispersion relation

Modified dispersion relation of GWs
has been tested by LIGO/Virgo collaboration. (1903.04467)

$$\delta\Phi_\alpha(f) = \mathrm{sign}(A_\alpha)\begin{cases} \dfrac{\pi D_L}{\alpha-1}\lambda_{A,\mathrm{eff}}^{\alpha-2}\left(\dfrac{f}{c}\right)^{\alpha-1}, & \alpha \neq 1 \\ \dfrac{\pi D_L}{\lambda_{A,\mathrm{eff}}}\ln\left(\dfrac{\pi G \mathcal{M}^{\mathrm{det}} f}{c^3}\right), & \alpha = 1 \end{cases}$$



α=0 ($A_\alpha > 0$) corresponds to the graviton mass
(although non-covariant theory)
$\Rightarrow \quad m_{\mathrm{graviton}} < 4.7 \times 10^{-23}$ eV

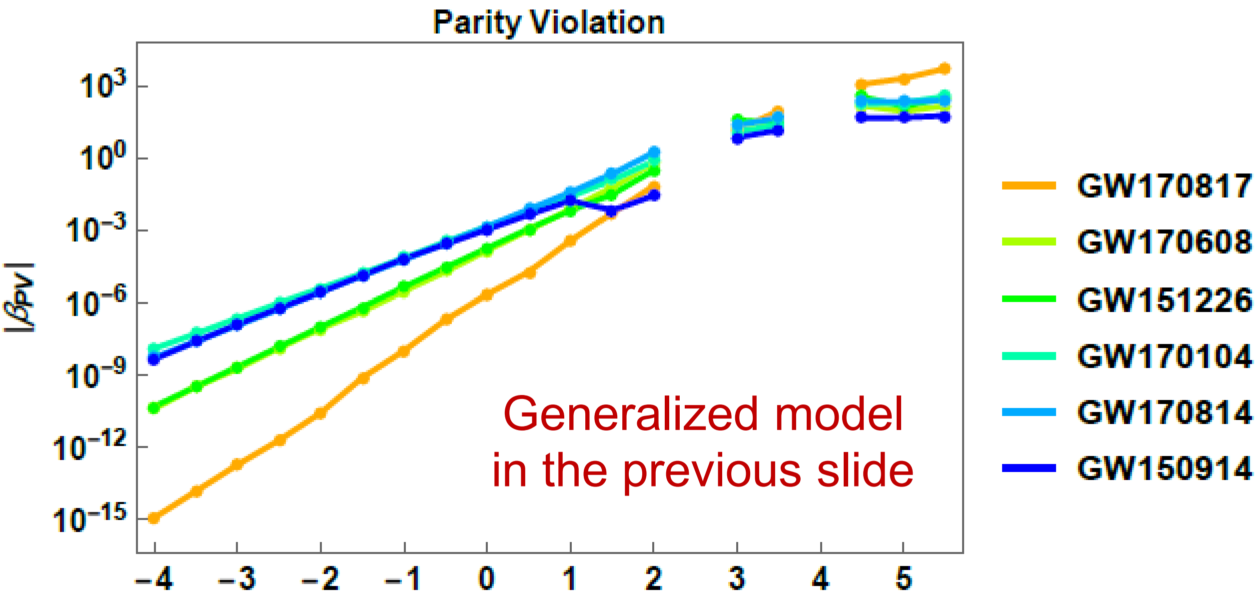
How about the parity violating cases?
Right and left-handed gravitons propagate differently.

$$\mathbf{h}^{(L,R)} = \frac{1}{\sqrt{2}}\left(\mathbf{h}^{(+)} + i\mathbf{h}^{(\times)}\right)$$

Result of application of the test to the real data

(K. Yamada and TT, PTEP 2020 093E01, arXiv:2006.11086)

$$\Psi(f) \to \Psi_{GR}(f) + \lambda_{L,R} \sum_i \beta_i u^{b_i}$$



The obtained upper bound magnitude is similar to non-parity violating case.

$$|\gamma - \delta| < 10^{-11}\,\mathrm{km} \quad \Rightarrow \quad |\gamma - \delta| < 10^{-18}\,\mathrm{km}$$

Constraint from $|c_T - 1| < 10^{-15}$

(Nishizawa, Kobayashi 1809.00815)

Generalized action including a term with the factor

$\varepsilon^{\alpha\beta\mu\nu} R_{\alpha\beta\gamma\delta} R_{\mu\nu\rho\sigma}$ and a scalar field.

$\phi\varepsilon^{\alpha\beta\mu\nu} R_{\alpha\beta\gamma\delta} R_{\mu\nu}{}^{\gamma\delta}$: Chern-Simons gravity is a typical example

Quadratic action (Nishizawa, Kobayashi 1809.00815)

$$L_{PV} = \frac{1}{4}\left[\gamma(t)\varepsilon^{ijk}\dot{h}_{il}\,\partial_j \dot{h}_k{}^l + \frac{\delta(t)}{a^2(t)}\varepsilon^{ijk}\partial^2 h_{il}\,\partial_j h_k{}^l\right]$$

Action for the Chern-Simons: $\gamma(t) = \delta(t)$

$$\sqrt{-g}\left(L_{GR} + L_{PV}\right) = a^2(t)\left[\underline{1 - k\lambda_{L,R}\gamma(t)}\right]\left[h'^2_{L,R} - k^2 h^2_{L,R}\right]$$

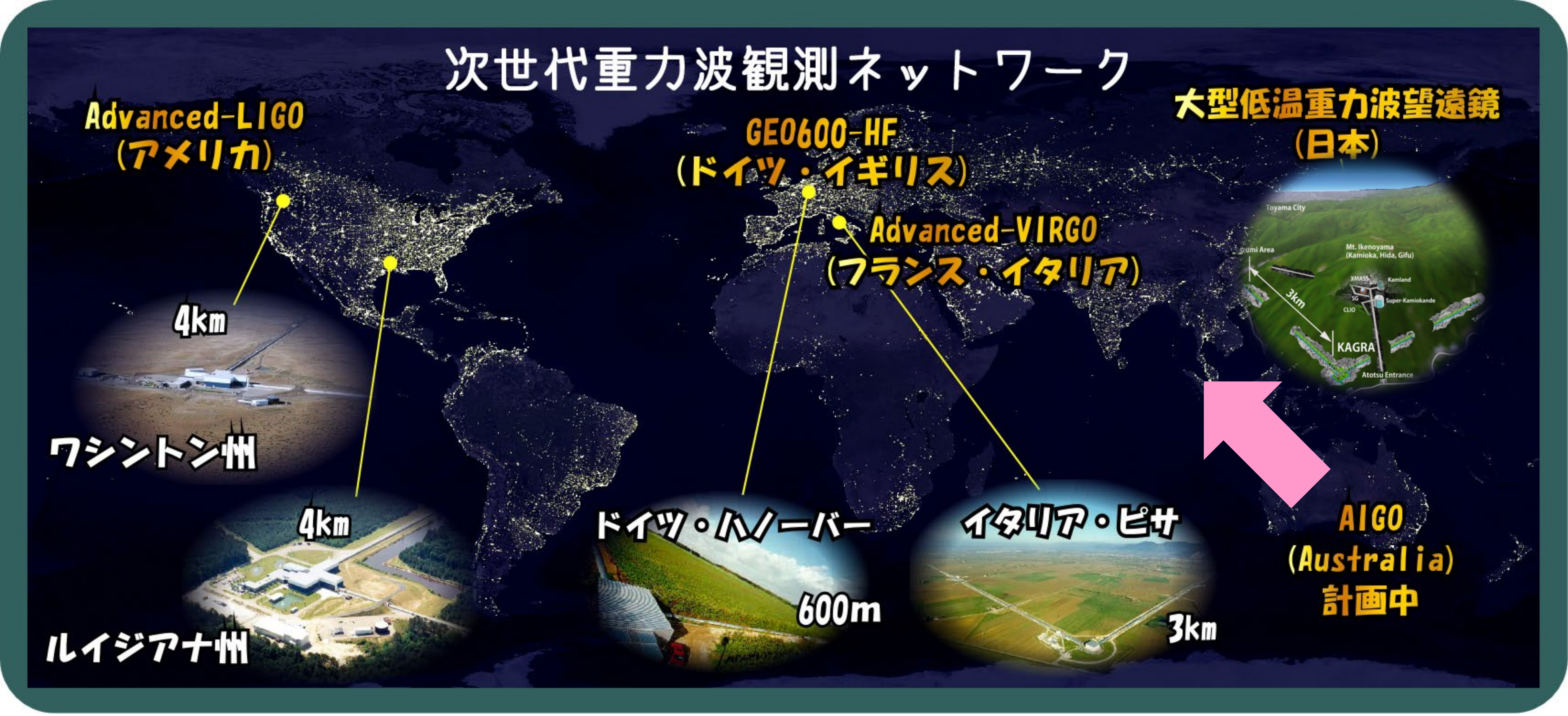
Large modification of amplitude occurs when this factor change in time: $a(t)\sqrt{1 - k\lambda_{L,R}\gamma(t)}\,h_{L,R} \sim \mathrm{const.}$

Remark) At a slightly higher frequency, negative energy states appear.

In more general models dispersion relation becomes

$$\omega^2 + \left(2aH - \lambda_{L,R}k\gamma'\right)i\omega - \left(1 + \lambda_{L,R}k(\gamma - \delta)\right)k^2 \approx 0$$

In general, gravitational wave phase is also modified.



Why do we need many similar detectors in the world ?

Difference in the arrival time

Direction of the source

EM follow-up observations

Sensitivity to each polarization mode depends on the antenna direction

Search for extra-polarization modes

Can we detect extra-polarization modes,
even if we assume some exotic extended gravity models? (arXiv:2304.14430)

Propagation process:

Case with the propagation direction $\boldsymbol{k}$ is in z-direction)

$$e^{+}_{ij} = \hat{e}_{x,i}\hat{e}_{x,j} - \hat{e}_{y,i}\hat{e}_{y,j}\,, \qquad e^{\times}_{ij} = \hat{e}_{x,i}\hat{e}_{y,j} + \hat{e}_{y,i}\hat{e}_{x,j}\,,$$
$$e^{x}_{ij} = \hat{e}_{x,i}\hat{e}_{z,j} + \hat{e}_{z,i}\hat{e}_{x,j}\,, \qquad e^{y}_{ij} = \hat{e}_{y,i}\hat{e}_{z,j} + \hat{e}_{z,i}\hat{e}_{y,j}\,,$$
$$e^{b}_{ij} = \hat{e}_{x,i}\hat{e}_{x,j} + \hat{e}_{y,i}\hat{e}_{y,j}\,, \qquad e^{l}_{ij} = \sqrt{2}\;\hat{e}_{z,i}\hat{e}_{z,j}\,.$$

$$h^{I}_{ij} = \sum_{A} h^{I}_{A} e^{A}_{ij}\,,$$

independently propagating modes, assuming slowly varying non-trivial background.

$$h_{ij} = \sum_{I} \phi_I h^{I}_{ij} + (\mathrm{c.c.})$$

$$\Rightarrow \quad S = \int d^4x \frac{\sqrt{-g}}{2} \sum_{I} \left(-g^{\mu\nu}\partial_\mu \phi^{*I}\partial_\nu \phi^{I}\right)$$

Quadratic action take this standard form by choosing $h^{I}{}_{A}$ appropriately under the condition that the propagation speed is unity.

$\Rightarrow$ Although the amplitude in terms of h_{ij} may change, the energy flux is not changed at all as long as the waveform is not significantly modified.

Detection process:

Case with an extra polarization exists)

We assume

1) no background anisotropies in solar system in gravitational sector and

2) the presence of an extra scalar propagating mode.

$$h_{ij} = \sum_{I=+,\times,s} \psi_I \hat{h}^I_{ij}$$

$$\hat{h}^+_{ij} = e^+_{ij}\,, \qquad \hat{h}^\times_{ij} = e^\times_{ij}\,,$$

$$\hat{h}^s_{ij} = \mathcal{A}_b e^b_{ij} + \mathcal{A}_l e^l_{ij}\,,$$

$$S_{\rm int} = \int d^4x \frac{\sqrt{-g}}{2} h_{\mu\nu} T^{\mu\nu}$$

$$h_{ij}(x) = \int d^4k\, e^{ik_\mu x^\mu} \tilde{G}_{ijkl}(k) \tilde{T}^{kl}(k)$$

$$= -\sum_I \frac{1}{4} \int d^4k\, e^{ik_\mu x^\mu} \frac{h^{*I}_{ij} h^I_{kl} + (\mathrm{c.c})}{\omega^2 - |\boldsymbol{k}|^2} \tilde{T}^{kl}(k)$$

Contribution of the tensor modes

$$h^{(T)}_{00} = \frac{M}{8\pi r}\,, \qquad h^{(T)}_{0i} = 0\,, \qquad h^{(T)}_{ij} = \frac{M}{8\pi r}\delta_{ij}\,.$$

Adding scalar contribution

$$\frac{\gamma - 1}{2} := \frac{(\delta^{ij} h_{ij}/3) - h_{00}}{h_{00}} \approx -2\Re\left[\left(\mathcal{A}_b - \frac{1}{\sqrt{2}}\mathcal{A}_l\right)\left(\mathcal{A}^*_b - \sqrt{2}\mathcal{A}^*_l\right)\right] < 1.2 \times 10^{-5}$$

Presence of scalar mode is observationally allowed only for fine tuned case.

Even in that case PN correction to the Newtonian potential will constrain the amplitude of the scalar mode. ($|\beta - 1| < 3 \times 10^{-3}$)

Detection process:

Case with background anisotropies)

We assume 1) no extra propagating mode and

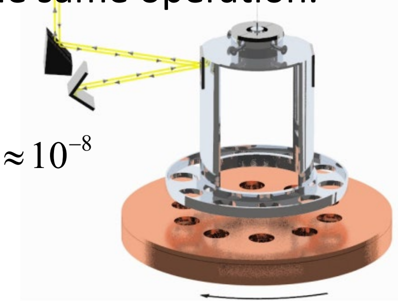
2) the background spatial vector V^i and tensor W^{ij} to the linear order

(a) $V^k e^{(T)}_{k(i}k_{j)}, V^k k_k e^{(T)}_{ij}$ (b) $W^{kl}e^{(T)}_{kl}k_ik_j, W^{kl}k_kk_le^{(T)}_{ij}, k_kW^k{}_{(i}e^{(T)}_{j)l}k^l$ (c) $k^2\delta_{ij}W^{kl}e^{(T)}_{kl}$ (d) $kW^k{}_{(i}e^{(T)}_{j)k}$

$e^{(T)}_{ij} = e^{(+)}_{ij}$ or $e^{(\times)}_{ij}$ + mode and x mode should be modified by the same operation.

(a)-terms give correction to the Newtonian potential

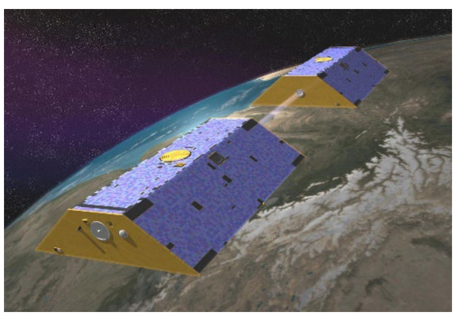
$h_{00} \approx V^k\partial_k \frac{M}{r}$ Shift of the center of mass $|V|k_{GW} < 1\mathrm{mm}\times 200\mathrm{Hz} \approx 10^{-8}$



Eöt-Wash

(b)-terms give correction to the Newtonian potential

$h_{00} \approx W^{kl}\partial_k\partial_l \frac{M}{r}$ The earth has fixed quadrupole moments Q



GRACE

$$\frac{\delta r}{r} \approx \frac{\delta\phi}{\phi} \approx \frac{Q}{M_\oplus R_\oplus^2} \approx \frac{|W|}{R_\oplus^2}, \quad \delta l_{\max} \approx \delta v P \approx \frac{\delta r}{r} v P = \frac{|W|}{R_\oplus},$$

$$\delta l \approx \delta l_{\max}\frac{r_{sep}}{R_\oplus} = \frac{|W| r_{sep}}{R_\oplus}$$

$$|W|k_{GW}^2 < \frac{\sqrt{\langle \delta l^2\rangle}}{r_{sep}} k_{GW}^2 R_\oplus^2 \approx \frac{1\mathrm{mm}}{220\mathrm{km}}\left(\frac{200\mathrm{Hz}}{50\mathrm{Hz}}\right)^2 \approx 10^{-7}$$

(c)-term gives no correction to the Newtonian potential, but not detected by GW detectors.

$$(a)\ V^k e^{(T)}_{k(i}k_{j)}, V^k k_k e^{(T)}_{ij} \quad (b)\ W^{kl} e^{(T)}_{kl} k_i k_j, W^{kl} k_k k_l e^{(T)}_{ij}, k_k W^k_{\ (i} e^{(T)}_{j)l} k^l \quad (c)\ k^2 \delta_{ij} W^{kl} e^{(T)}_{kl} \quad (d)\ k W^k_{\ (i} e^{(T)}_{j)k}$$

(d)-term gives no correction to the Newtonian potential and can be detected as the breathing mode by GW detectors.

$$h_{0i} \approx W_i^{\ j} \partial_j \frac{M}{r} \Rightarrow \frac{\delta r}{r} \approx \frac{\delta\phi}{\phi}\frac{v}{c} \approx \frac{|W|}{R_\oplus}\frac{v}{c}, \qquad \delta l_{\max} \approx \delta v P \approx \frac{\delta r}{r} v P = |W|\frac{v}{c},$$

$$\delta l \approx \delta l_{\max} \frac{r_{sep}}{R_\oplus} = |W| \frac{r_{sep}}{R_\oplus}\frac{v}{c}$$

$$\Rightarrow |W| k_{GW} < \frac{\sqrt{\langle \delta l^2 \rangle}}{r_{sep}} k_{GW} R_\oplus \left(\frac{v}{c}\right)^{-1} \approx \frac{1\mathrm{mm}}{220\mathrm{km}}\left(\frac{200\mathrm{Hz}}{50\mathrm{Hz}}\right) 10^4 \approx 10^{-3}$$

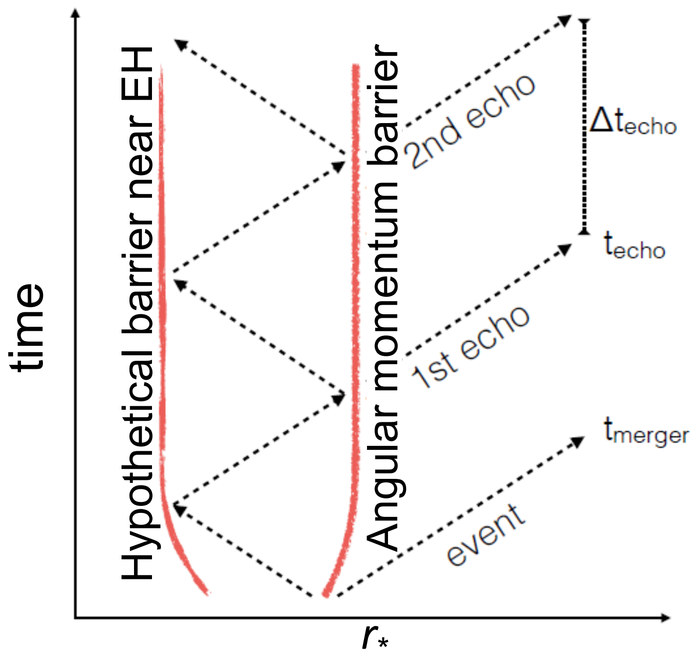
The constraint is weaker owing to the reduction factor $v/c \simeq 10^{-4}$.

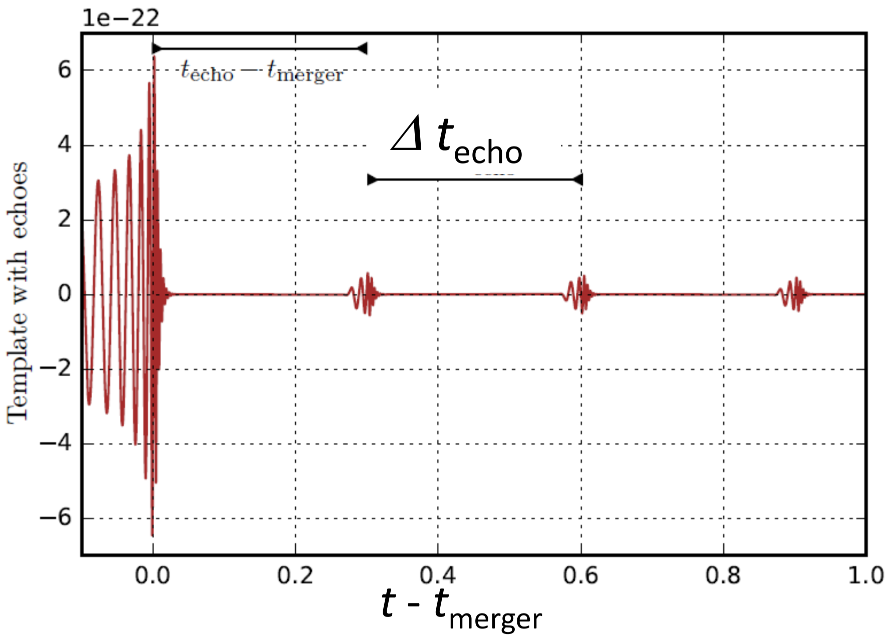
To conclude, extra GW polarizations induced by the hypothetical hidden local anisotropic background are hardly detected, as long as simultaneous detection is assumed using GR templates.

Black Hole Echoes

There was a claim that echoes might have been detected in the LIGO GW events.

(Abedi et al, arXiv:1612.00266)





1) Reflecting boundary at the Planck distance from the horizon
2) Slowly decaying echoes of the same waveform with overall phase shift π.
3) Echo period Δt_{echo} is the one predicted by the angular momentum barrier and the reflection wall at the Planck distance from the horizon.

   $\rightarrow 3\sigma$ detection of signal

We considered more appropriate template will increase the significance (arXiv:1704.07175)

Perturbation Equation of Kerr BH (Sasaki-Nakamura) [$l=m=2$ mode only]

$$\frac{d^2Y}{dr_*^2} + V_{SN}Y = 0 \quad \Rightarrow \quad Y = \begin{cases} \exp(-i\omega r_*) & (r_* \to \infty) \\ Y_{up}\exp(-ikr_*) + Y_{down}\exp(ikr_*) & (r_* \to -\infty) \end{cases}$$

Reflection rate :

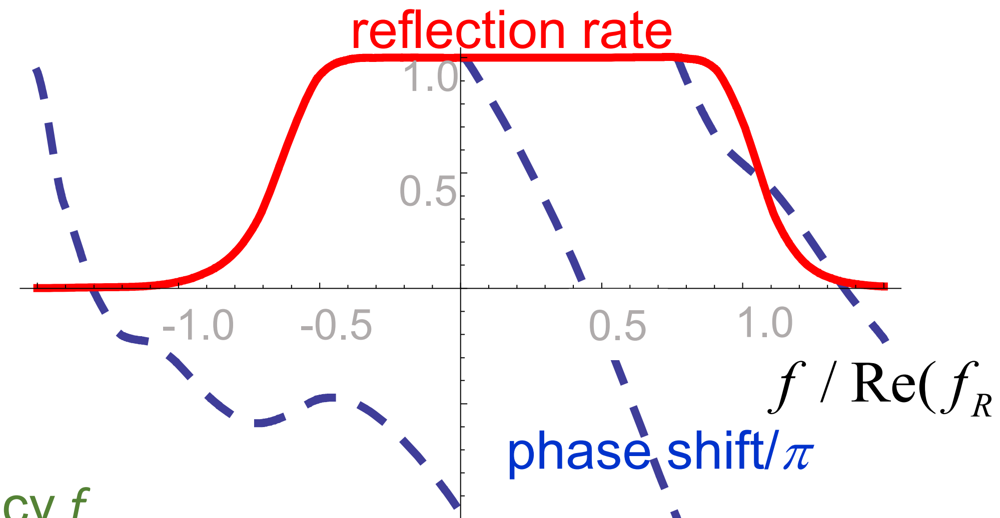
$$\sqrt{\mathrm{R}(f)} = \sqrt{C}\,\frac{|Y_{up}|}{|Y_{down}|}$$

$C \neq 1$ since V_{SN} is not real

Phase shift :

$$\phi(f) = \arg\left(\frac{Y_{up}}{Y_{down}}\right)?$$

depends on the frequency f.

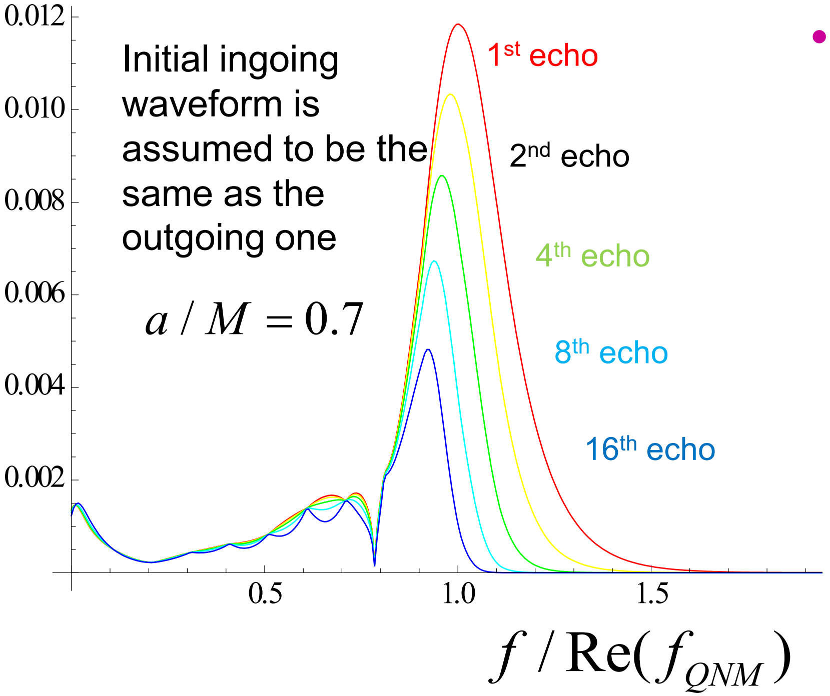


ϕ can depend on the nature of the near-horizon boundary even if we assume complete reflection.

However, $\phi(f)$ can be approximated by a linear fn. for a narrow range of f close to the QNM frequency f_R.

⇒ $\phi(f)$ can be absorbed by the time interval between echoes and the overall phase shift ϕ_0

Power spectrum changes at each reflection



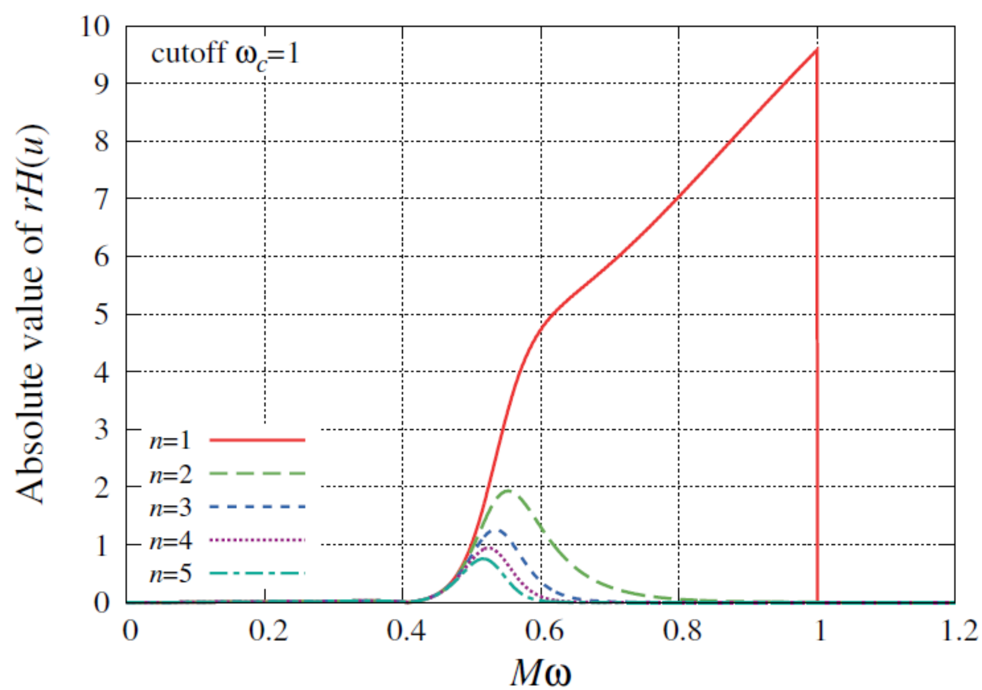
- Low frequency modes cannot penetrate the angular momentum barrier.

There is a different opinion. (1907.03091)

The waveform is totally different from the simple repetition of initial outgoing waveform used by the analyses by other groups.

Echo spectrum excited by a small particle falling into Kerr black hole.

$$a/M = 0.7,\ L_z = 0.9 L_{\mathrm{marginal}}$$



Early phase of echo can be much louder, especially at high frequencies.

(Sago and T.T. arXiv 2009.08086)

How does the significance of the echo signal change by taking into account this more physically motivated waveform?

No Signal!

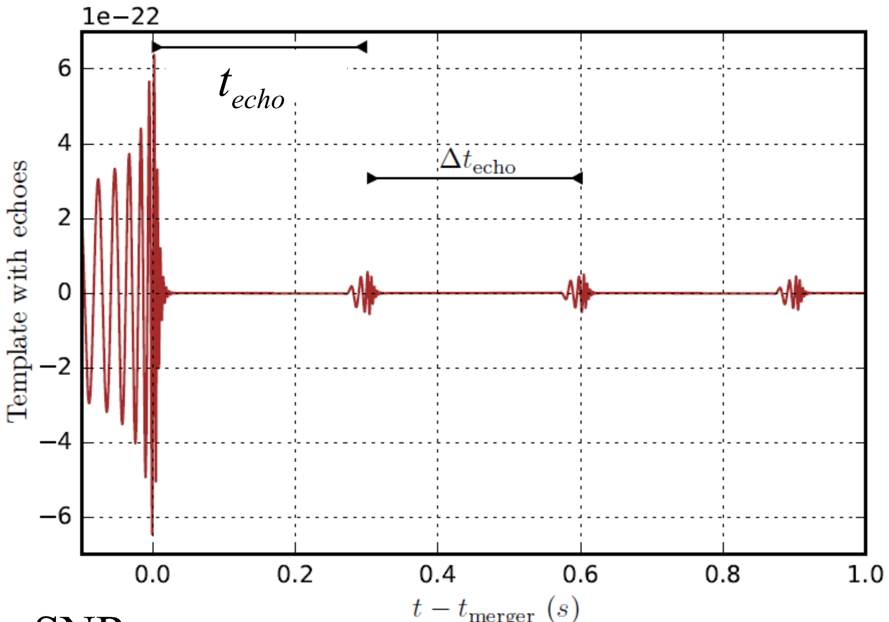
Analysis method

N. Uchikata, H. Nakano, T. Narikawa, N. Sago, H. Tagoshi, TT arXiv:1906.00838
N. Uchikata, et al., in preparation

We follow the basic strategy taken by Abedi et al. (2017)

Search the maximum signal-to-noise ratio (SNR) in the interval of $0.99 \le x \le 1.01$, varying the other parameters.

$$x \equiv \frac{t_{echo}}{\Delta t_{echo}}$$



Background estimation
Perform the same analysis for the neighboring 4096sec data, where no signal is expected.

p-value

$$p \equiv \frac{\#\,\text{of reference data with SNR} > \text{SNR}_{\text{event}}}{\#\,\text{of all reference data}}$$

If we find p << 1, there might be some true signal.

Results of our re-analysis

We performed reanalysis using our new template, which we think is more physically motivated, expecting the increase of significance.

List of p-values

	Simple repetition	Our template
GW150914	0.157	0.984
GW151012	0.047	0.882
GW170104	0.071	0.677
GW170608	0.079	0.488
GW170729	0.567	0.575
GW170814	0.024	0.472
GW170818	0.929	0.976
GW170823	0.055	0.315
average	0.241	0.671

$2.75\,\sigma$

The analysis using the template assuming simple repetition of the same waveform gives relatively small p-value, although p-value is large for a few events.

Expected improvement of the significance was not achieved.

Challenges on testing gravity

Motivation to consider gravity theories beyond GR

- Cosmological observations (DE/DM, Hubble/S_8 tension, etc.)
- Black hole information paradox
- Quantum gravity

Constructing gravitational wave templates in extended gravity scenarios

- Higher dimensional operator corrections (EDGB, CS).
- Low energy extra degrees of freedom that evades all the weak gravity tests (Screening mechanism, Parity odd, Very weakly interacting with matter).
- Modified nature of black hole horizon (ECO).

Challenges

- Theoretical predictions of the waveform for binary coalescence in extended gravity theories are very limited.
- Optimizing data analyses based on the theoretical inputs.
- For extreme mass-ratio inspirals, even GR templates are not ready yet.