EFT of scalar-tensor gravity with timelike scalar profile

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4. BH with timelike scalar profile
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6. Summary

Appx A. Generalized 2nd law
Appx B. de Sitter entropy bound

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Ref. arXiv: 2204.00228 w/ V.Yingcharoenrat
(arXiv: 2111.08119 w/ K.Aoki, M.A.Gorji & K.Takahashi)

Also Arkani-Hamed, Cheng, Luty and Mukohyama 2004 (hep-th/0312099)
Mukohyama 2005 (hep-th/0502189)

Collaborators

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INTRODUCTION

Why gravity beyond GR?

(GR : general relativity)

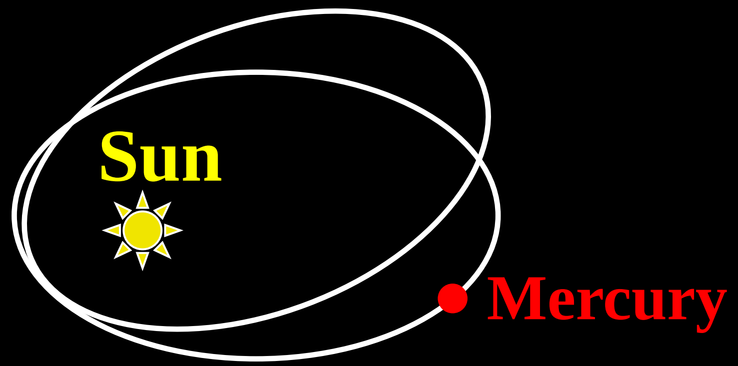
A motivation for IR modification

- Gravity at long distances
  Flattening galaxy rotation curves
  extra gravity
  Dimming supernovae
  accelerating universe

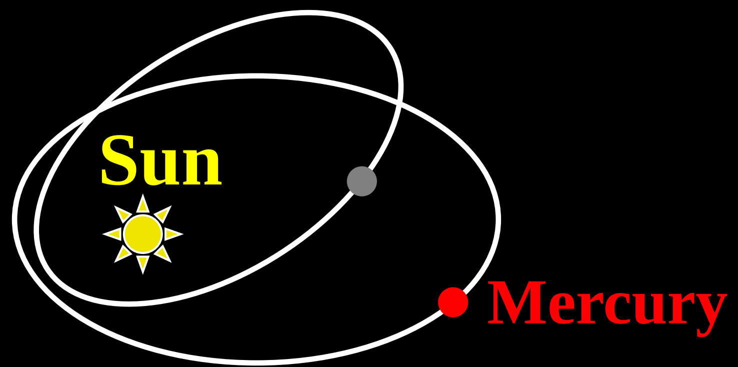
- Usual explanation: new forms of matter (DARK MATTER) and energy (DARK ENERGY).

Dark component in the solar system?

Precession of perihelion observed in 1800's…



which people tried to explain with a "dark planet", Vulcan,



But the right answer wasn't "dark planet", it was "change gravity" from Newton to GR.

Why gravity beyond GR?

- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.

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  Superstring, Horava-Lifshitz, etc.

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- Can we address **mysteries in the universe**?
  Dark energy, dark matter, inflation, big-bang singularity, cosmic magnetic field, etc.
- Help constructing a **theory of quantum gravity**?
  Superstring, Horava-Lifshitz, etc.
- Do we really **understand GR**?
  One of the best ways to understand something may be to break (modify) it and then to reconstruct it.
- …

EFT of scalar-tensor gravity
with timelike scalar profile

EFT ON MINKOWSKI BACKGROUND

Many gravity theories

- 3 check points
  “What are the physical d.o.f. ?”
  “How do they interact ?”
  “What is the regime of validity ?”
- If two (or more) theories give the same answers to the 3 questions above then they are the same even if they look different.
  → Effective Field Theory (EFT)
  as universal description

Scalar-tensor theories of gravity

- Metric $g_{\mu\nu}$ + scalar field ϕ
- Jordan (1955), Brans & Dicke (1961), Bergmann (1968), Wagoner (1970), …
- Most general scalar-tensor theory of gravity with 2^{nd} order covariant EOM: Horndeski (1974)
- DHOST theories beyond Horndeski: Langlois & Noui (2016)
- U-DHOST theories beyond DHOST: DeFelice, Langlois, Mukohyama, Noui & Wang (2018)
- All of them (and more) are universally described by an effective field theory (EFT)

EFT of inflation / dark energy = EFT of scalar-tensor gravity with timelike scalar profile

- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing Nambu-Goldstone boson

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Simplest : ghost condensation

ref. Arkani-Hamed, Cheng, Luty, Mukohyama 2004

Systematic construction of EFT of ghost condensation

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

Backgrounds characterized by

✧ $\left\langle \partial_\mu \phi \right\rangle \neq 0$ and timelike

✧Background metric is Minkowski.

⇨ $$\boxed{\begin{aligned} L_{eff} = L_{EH} + M^4 \Big\{ & \left(h_{00} - 2\dot{\pi}\right)^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 \\ & - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \Big\} \end{aligned}}$$

Gauge choice: $\phi(t,\vec{x}) = t.$ $\pi \equiv \delta\phi = 0$

(Unitary gauge)

Residual symmetry: $\vec{x} \to \vec{x}'(t,\vec{x})$

$\Rightarrow$ Write down most general action invariant under this residual symmetry.

($\Rightarrow$ Action for π: undo unitary gauge!)

Start with flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Under residual ξ^i

$$\delta h_{00} = 0, \delta h_{0i} = \partial_0 \xi_i, \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i$$

Action invariant under ξ^i

$$\begin{cases} (h_{00})^2 & \textbf{OK} \\ \cancel{(h_{0i})^2} & \\ K^2, K^{ij}K_{ij} & \textbf{OK} \end{cases}$$

Beginning at quadratic order, since we are assuming flat space is good background.

$$K_{ij} = \frac{1}{2}\left(\partial_0 h_{ij} - \partial_j h_{0i} - \partial_i h_{0j}\right)$$

$$\Rightarrow \quad L_{eff} = L_{EH} + M^4\left\{(h_{00})^2 - \frac{\alpha_1}{M^2}K^2 - \frac{\alpha_2}{M^2}K^{ij}K_{ij} + \cdots\right\}$$

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Action for π

$$\xi^0 = \pi \quad \begin{cases} h_{00} \to h_{00} - 2\partial_0 \pi \\ K_{ij} \to K_{ij} + \partial_i \partial_j \pi \end{cases}$$

$$\Rightarrow \quad \boxed{L_{eff} = L_{EH} + M^4 \left\{ (h_{00} - 2\dot{\pi})^2 - \frac{\alpha_1}{M^2}\left(K + \vec{\nabla}^2 \pi\right)^2 - \frac{\alpha_2}{M^2}\left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi\right)\left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi\right) + \cdots \right\}}$$

$$E \to rE$$

$$dt \to r^{-1} dt$$

$$dx \to r^{-1/2} dx$$

$$\pi \to r^{1/4} \pi$$

Make invariant $\Rightarrow \int dt d^3x \left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2\pi)^2}{M^2} + \cdots \right]$

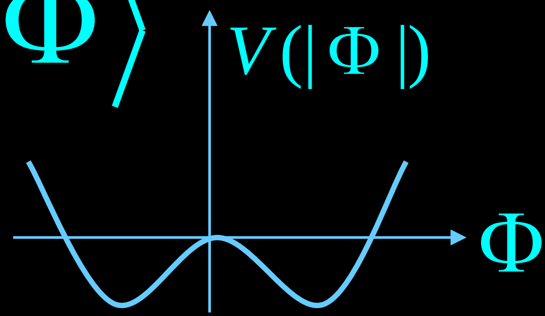
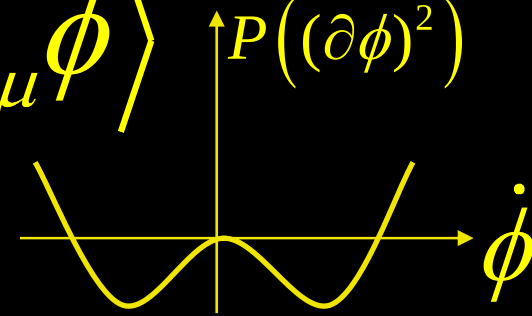
Leading nonlinear operator in infrared $\int dt d^3x \frac{\dot{\pi}(\nabla\pi)^2}{\tilde{M}^2}$

has scaling dimension 1/4. **(Barely) irrelevant**

$\Rightarrow$ Good low-E effective theory

Robust prediction

e.g. Ghost inflation [Arkani-hamed, Creminelli, Mukohyama, Zaldarriaga 2004]

	Higgs mechanism	***Ghost condensate*** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V’=0, V’’>0	P’=0, P’’>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

EFT of ghost condensation =
EFT of scalar-tensor gravity with timelike scalar profile on Minkowski background

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

Backgrounds characterized by

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EFT ON COSMOLOGICAL BACKGROUND

Extension to FLRW background = EFT of inflation/dark energy

Creminelli, Luty, Nicolis, Senatore 2006
Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007

- Action invariant under $x^i \rightarrow x^i(t,x)$
- Ingredients
  $g_{\mu\nu}, g^{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu,$ t & its derivatives
- 1st derivative of t

$$\partial_\mu t = \delta^0_\mu \qquad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}} = \frac{\delta^0_\mu}{\sqrt{-g^{00}}}$$

$$g^{00} \qquad h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$

- 2nd derivative of t

$$K_{\mu\nu} \equiv h^\rho_\mu \nabla_\rho n_\nu$$

Unitary gauge action

$$\boxed{I = \int d^4x\sqrt{-g}L(t, \delta^0_\mu, K_{\mu\nu}, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, R_{\mu\nu\rho\sigma})}$$

derivative & perturbative expansions

$$\begin{aligned} I &= M_{Pl}^2 \int dx^4\sqrt{-g}\left[\frac{1}{2}R + c_1(t) + c_2(t)g^{00}\right. \\ &\quad \left. + L^{(2)}(\tilde{\delta}g^{00}, \tilde{\delta}K_{\mu\nu}, \tilde{\delta}R_{\mu\nu\rho\sigma}; t, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu)\right] \end{aligned}$$

$$\begin{aligned} L^{(2)} &= \lambda_1(t)(\tilde{\delta}g^{00})^2 + \lambda_2(t)(\tilde{\delta}g^{00})^3 + \lambda_3(t)\tilde{\delta}g^{00}\tilde{\delta}K^\mu_\mu \\ &\quad + \lambda_4(t)(\tilde{\delta}K^\mu_\mu)^2 + \lambda_5(t)\tilde{\delta}K^\mu_\nu\tilde{\delta}K^\nu_\mu + \cdots \end{aligned}$$

NG boson

- Undo unitary gauge $t \to \tilde{t} = t - \pi(\tilde{t}, \vec{x})$

$$
\begin{aligned}
H(t) &\to H(t+\pi), \quad \dot{H}(t) \to \dot{H}(t+\pi), \\
\lambda_i(t) &\to \lambda_i(t+\pi), \quad a(t) \to a(t+\pi), \\
\delta^0_\mu &\to (1+\dot{\pi})\delta^0_\mu + \delta^i_\mu \partial_i \pi,
\end{aligned}
$$

- NG boson in decoupling (subhorizon) limit

$$
\boxed{
\begin{aligned}
I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \Bigg\{ & -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \\
& -\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta}K, \tilde{\delta}R} \Bigg\}
\end{aligned}
}
$$

$$
\frac{1}{c_s^2} = 1 - \frac{4\lambda_1}{\dot{H}}, \quad c_3 = c_s^2 - \frac{8c_s^2\lambda_2}{-\dot{H}}\left(\frac{1}{c_s^2} - 1\right)^{-1}
$$

- Sound speed

c_s : speed of propagation for modes with $\omega \gg H$

$\omega^2 \simeq c_s^2 \dfrac{k^2}{a^2}$ for $\pi \sim A(t)\exp\left(-i\int \omega dt + i\vec{k}\cdot\vec{x}\right)$

Application: non-Gaussinity of inflationary perturbation $\zeta = -H\pi$

$$I_\pi = M_{Pl}^2 \int dt d^3\vec{x}\, a^3 \left\{ -\frac{\dot{H}}{c_s^2}\left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2}\right) \right.$$

$$\left. -\dot{H}\left(\frac{1}{c_s^2} - 1\right)\left(\frac{c_3}{c_s^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) + O(\pi^4, \tilde{\epsilon}^2) + L^{(2)}_{\tilde{\delta}K, \tilde{\delta}R} \right\}$$

2 types of 3-point interactions

→ power spectrum

$$P_\zeta(\vec{k}) = \frac{\Delta}{k^3}, \quad \Delta = \left.\frac{H^4}{-4M_{Pl}^2 \dot{H} c_s}\right|_{c_s k \simeq aH}$$

→ non-Gaussianity

$$\langle \zeta_{\vec{k}_1}(t)\, \zeta_{\vec{k}_2}(t)\, \zeta_{\vec{k}_3}(t) \rangle = (2\pi)^3 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_\zeta$$

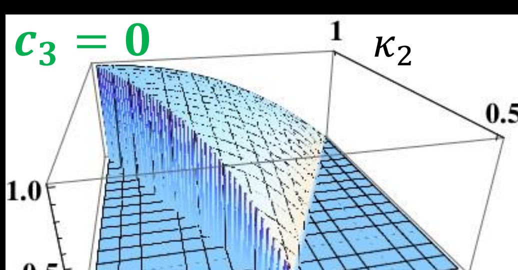
c_s^2 → size of non-Gaussianity

$$k^6 B_\zeta|_{k_1=k_2=k_3=k} = \frac{18}{5}\Delta^2\left(f_{NL}^{\dot{\pi}(\partial_i\pi)^2} + f_{NL}^{\dot{\pi}^3}\right)$$

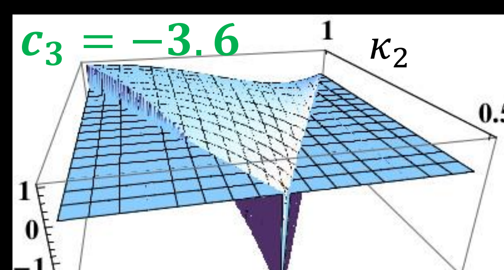
$$f_{NL}^{\dot{\pi}(\partial_i\pi)^2} = \frac{85}{324}\left(1 - \boxed{\frac{1}{c_s^2}}\right) \qquad f_{NL}^{\dot{\pi}^3} = \frac{5\boxed{c_3}}{81}\left(1 - \boxed{\frac{1}{c_s^2}}\right) \quad \propto \frac{1}{c_s^2} \text{ for small } c_s^2$$

c_3 → shape of non-Gaussianity

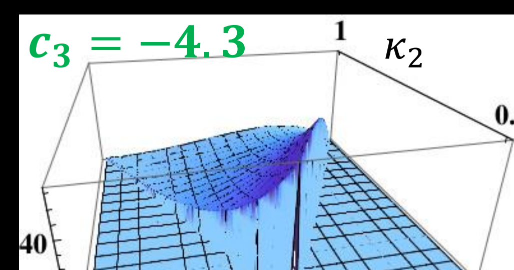
plots of $B_\zeta(k, \kappa_2 k, \kappa_3 k)/B_\zeta(k,k,k)$



Prototype of the equilateral shape



Prototype of the orthogonal shape



Linear combination of the two shapes

Summary so far

- Ghost condensation universally describes all scalar-tensor theories of gravity with timelike scalar profile on Minkowski background.
- Extension of ghost condensation to FLRW backgrounds results in the EFT of inflation/DE.
- These EFTs provide universal descriptions of all scalar-tensor theories of gravity with timelike scalar profile on each background, including Horndeski theory, DHOST theory, U-DHOST theory, and more.

EFT of scalar-tensor gravity
with timelike scalar profile

BLACK HOLE WITH TIMELIKE SCALAR PROFILE

- Cosmology and black holes (BHs) play as important roles in gravitational physics as blackbody radiation and hydrogen atoms did in quantum mechanics.
- In cosmology a time-dependent scalar field can act as dark energy (DE), while BHs serve as probes of strong gravity. We then hope to probe the scalar field DE by astrophysical BHs.
- This would require the scalar field profile to be timelike near BH. Otherwise, contours of the scalar field would become ill-defined.
- Are there BH solution with timelike scalar profile?

Stealth solutions in k-essence

Mukohyama 2005

- Action in Einstein frame

$$I=\int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R+P(X)\right]\qquad X=-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

- EOMs

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}P'(X)g^{\mu\nu}\partial_\nu\phi\right)=0$$

$$M_{\rm Pl}^2G_{\mu\nu}=2P'(X)\partial_\mu\phi\partial_\nu\phi+P(X)g_{\mu\nu}$$

- Stealth sol with $X = X_0$, where $P'(X_0)=0$

$$G_{\mu\nu}=\Lambda_{\rm eff}g_{\mu\nu}\qquad \Lambda_{\rm eff}=P(X_0)/M_{\rm Pl}^2$$

- $X = X_0$ ($\neq 0$)

$\Longleftrightarrow$ $u^\mu=g^{\mu\nu}\partial_\nu\phi$ defines geodesic congruence

$$(u^\nu\nabla_\nu u^\mu=-\nabla^\mu X/2=0)$$

$\Longleftrightarrow$ $\phi/\sqrt{|X_0|}$ **defines Gaussian normal coord.**

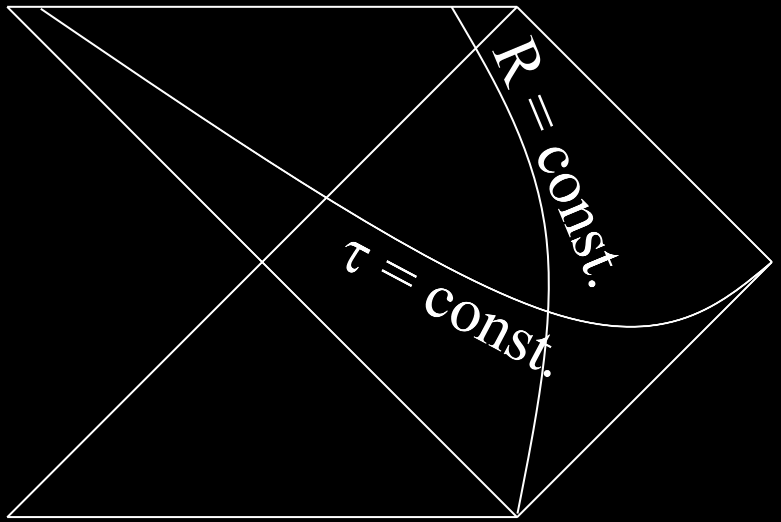
Stealth solutions in k-essence

Mukohyama 2005

- Any metric locally admits Gaussian normal coord.
- If P'(X) has a real root X_0 then any vacuum GR sol with $\Lambda_{\rm eff} = P(X_0)/M_{\rm Pl}^2$ locally leads to a stealth sol.
- Schwarzshild metric admits a "globally" well-behaved Gaussian normal coord. (Lemeitre reference frame)

$$g_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + \frac{r_g dR^2}{r(\tau,R)} + r^2(\tau,R)d\Omega^2$$

$$r(\tau,R) = \left[\frac{3}{2}\sqrt{r_g}(R-\tau)\right]^{2/3}$$



- Stealth Schwarzschild solution with $\phi = \sqrt{X_0}\tau$, if P'(X) has a positive root X_0 and if $\Lambda_{\rm eff}$ is canceled by $\Lambda_{\rm bare}$

Stealth solutions with $\phi = qt+\psi(r)$

- Schwarzschild in k-essence (Mukohyama 2005)
- Schwarzschild-(A)dS in Horndeski theory (Babichev & Charmousis 2013, Kobayashi & Tanahashi 2014) Schwarzshild-(A)dS in DHOST (Ben Achour & Liu 2019, Motohashi & Minamitsuji 2019)
- Kerr-(A)dS in DHOST (Charmousis & Crisotomi & Gregory & Stergioulas 2019)
- However, perturbations around most of those stealth solutions are infinitely strongly coupled (de Rham & Zhang 2019) . This means the solutions cannot be trusted.
- Approximately stealth solution in ghost condensate does not suffer from strong coupling (Mukohyama 2005). Why?

Origin of strong coupling

- EFT around stealth Minkowski sol. (= ghost condensate) → universal dispersion relation without the usual k^2 term

$$\boxed{\omega^2 = \alpha k^4 / M^2}$$

- For $\alpha = O(1)$ (>0), EFT is weakly coupled all the way up to ~M.
- If eom's for perturbations are strictly 2nd order (as in DHOST) then $\alpha = 0$ and the dispersion relation loses dependence on k
→ strong coupling
- [For $\omega^2 = c_s^2 k^2$, strong coupling @ E~$c_s^{7/4} M$]

Strong coupling scales

- EFT of inflation/DE in decoupling limit

$$S_\pi = M_{\rm Pl}^2 \int dt d^3\vec{x}\, a^3 \left[-\frac{\dot{H}}{c_{\rm s}^2}\left(\dot{\pi}^2 - c_{\rm s}^2 \frac{(\partial_i \pi)^2}{a^2}\right) - \dot{H}\left(\frac{1}{c_{\rm s}^2} - 1\right)\left(\frac{c_3}{c_{\rm s}^2}\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) + \mathcal{O}(\pi^4, \tilde{\epsilon}^2) + \mathcal{L}^{(2)}_{\delta K, \delta R} \right]$$

$$\frac{1}{c_{\rm s}^2} = 1 + \frac{4\lambda_1}{-\dot{H}}, \quad c_3 = c_{\rm s}^2 - \frac{8c_{\rm s}^2\lambda_2}{-\dot{H}}\left(\frac{1}{c_{\rm s}^2} - 1\right)^{-1}$$

- If $c_s^2 \simeq$ const is not too small, $\mathcal{L}^{(2)}_{\tilde{\delta K}, \tilde{\delta R}}$ can be ignored. We further assume $0 < c_s < 1$.

$$S_\pi = \int dt d^3\vec{\tilde{x}}\, a^3 (c_{\rm s}\epsilon M_{\rm Pl}^2 H^2)\left[\dot{\pi}^2 - \frac{(\tilde{\partial}_i\pi)^2}{a^2} + \left(\frac{1}{c_{\rm s}^2} - 1\right)\dot{\pi}\left(c_3\dot{\pi}^2 - \frac{(\tilde{\partial}_i\pi)^2}{a^2}\right) + \cdots\right]$$

$$\vec{x} = c_{\rm s}\vec{\tilde{x}}$$

$$\dot{\pi}^2 \sim \frac{(\tilde{\partial}_i\pi)^2}{a^2} \sim \frac{E^4}{c_{\rm s}\epsilon M_{\rm Pl}^2 H^2} \qquad \left(\frac{1}{c_{\rm s}^2} - 1\right)|\dot{\pi}|\Big|_{E = E_{\rm cubic}} \sim \frac{1}{\max[|c_3|, 1]}$$

$$E_{\rm cubic} \lesssim \frac{(c_{\rm s}^5 \epsilon M_{\rm Pl}^2 H^2)^{1/4}}{\sqrt{1 - c_{\rm s}^2}} \to 0 \qquad \left(c_{\rm s}^5\epsilon/(1 - c_{\rm s}^2)^2 \to 0\right)$$

A solution: scordatura

Motohashi & Mukohyama 2019

- Detuning of degeneracy condition recovers $\omega^2 = \alpha k^4/M^2$ and uplifts the strong coupling scale to ~$|\alpha|^{7/2}M$. If the amount of detuning is small enough then an apparent ghost is heavy enough to be integrated out.
- Scordatura = weak and controlled detuning of degeneracy condition
- Scordatura DHOST realizes ghost condensation near stealth solutions while it behaves as DHOST away from them.

Strong coupling scales

- De Sitter limit = small c_s^2 limit

$$S_\pi = M_{\rm Pl}^2 \int dt d^3\vec{x}\, a^3 \left[4\lambda_1 \left(\dot\pi^2 - c_{\rm s}^2 \frac{(\partial_i \pi)^2}{a^2} - \dot\pi \frac{(\partial_i\pi)^2}{a^2} \right) + 4(\lambda_1 - 2\lambda_2)\dot\pi^3 + \lambda_3 \left(H - \frac{\partial_j^2 \pi}{a^2} \right) \frac{(\partial_i\pi)^2}{a^2} + (\lambda_4 + \lambda_5) \frac{(\partial_i^2 \pi)^2}{a^4} + \cdots \right]$$

$$\lambda_1 = \frac{M^4}{8M_{\rm Pl}^2}, \quad \lambda_3 = \frac{M^3\beta}{2M_{\rm Pl}^2}, \quad \lambda_4 = -\frac{M^2(\alpha+\gamma)}{2M_{\rm Pl}^2}, \quad \lambda_5 = \frac{M^2\gamma}{2M_{\rm Pl}^2}$$

$$S_\pi = \frac{M^4}{2} \int dt d^3\vec{x}\, a^3 \left[\dot\pi^2 - c_{\rm s}^2 \frac{(\partial_i\pi)^2}{a^2} - \dot\pi \frac{(\partial_i\pi)^2}{a^2} - \frac{\alpha}{M^2} \frac{(\partial_i^2\pi)^2}{a^4} + \frac{\beta}{M} \left(H - \frac{\partial_j^2\pi}{a^2} \right) \frac{(\partial_i\pi)^2}{a^2} + \cdots \right]$$

$$E^{-1} p^{-3} M^4 (E\pi)^2 \sim 1 \quad \Rightarrow \quad \pi \sim \frac{E^{3/2}}{p^{1/2} M^2}$$

$$\left. \frac{E\pi p^2}{E^2} \right|_{E=E_{\rm cubic}} \sim 1 \quad \Rightarrow \quad \left. \left(\frac{p}{E} \right)^{7/4} \frac{E}{M} \right|_{E=E_{\rm cubic}} \sim 1$$

$$\frac{\omega^2}{M^2} = \alpha \frac{k^4}{M^4 a^4} \quad \text{for} \quad \max\left[c_{\rm s}^2, |\beta| \frac{H}{M} \right] \ll |\alpha| \frac{k^2}{M^2 a^2} \ll 1$$

$$\Rightarrow \quad E_{\rm cubic} \simeq |\alpha|^{7/2} M$$

Approximately stealth BH in ghost condensate

Mukohyama 2005

- Two time scales: $t_{BH} << t_{GC} \sim M_{Pl}^2/M^3$
- For $t_{BH} << t << t_{GC}$, a usual BH sol is a good approximation → approximately stealth

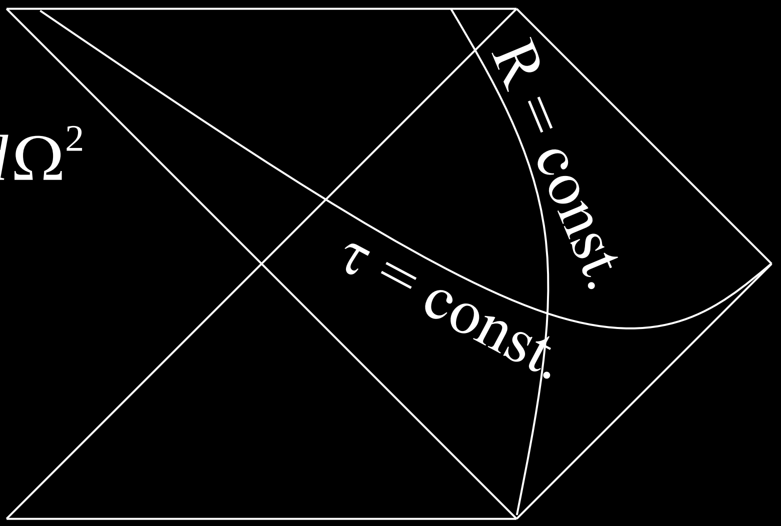
Schwarzschild metric:

$$g_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + \frac{r_g dR^2}{r(\tau,R)} + r^2(\tau,R)d\Omega^2$$

$$r(\tau,R) = \left[\frac{3}{2}\sqrt{r_g}(R-\tau)\right]^{2/3}$$

$$E = -\xi^\mu p_\mu \qquad \xi^\mu = \partial_\tau + \partial_R$$

$\phi = M^2\tau$ ➡ Exact sol in the absence of higher derivative terms



Approximately stealth BH in ghost condensate

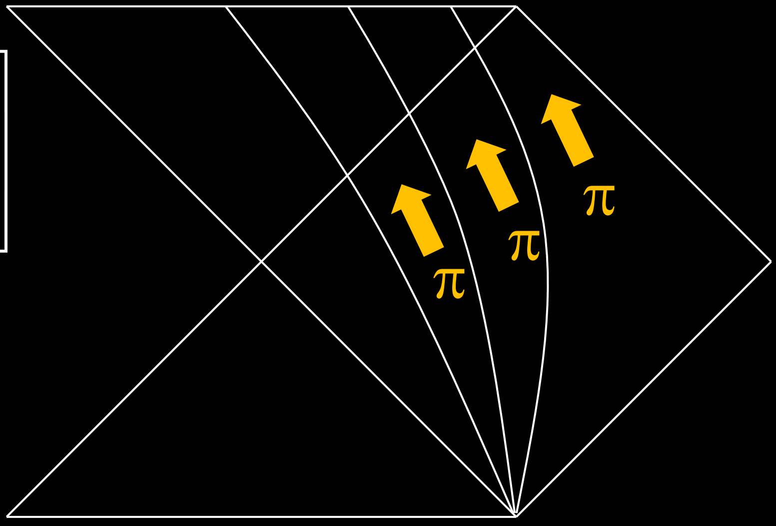
Mukohyama 2005; Cheng, Luty, Mukohyama and Thaler 2006

- A tiny tadpole due to higher derivative terms is canceled by extremely slow time-dependence.
- As a result, $\pi = \delta\phi$ starts accreting gradually.
- XTE J1118+480 (M_{bh}~$7M_{sun}$, r~$3R_{sun}$, t~240Myr or 7 Gyr) ➡ $M<10^{12}$GeV much weaker than $M<100$GeV

$$M_{bh} = M_{bh0} \times \left[1 + \frac{9\alpha M^2}{4M_{Pl}^2} \left(\frac{3M_{Pl}^2 v}{4M_{bh0}} \right)^{2/3} \right]$$

v : advanced null coordinate

α : coefficient of h.d. term



Summary of stealth BH with timelike scalar profile

- If we want to learn something about scalar field DE from BH then we need to consider BH solutions with timelike scalar profile
- Stealth solutions = backgrounds with GR metric and non-trivial scalar profile → examples of BH solutions with timelike scalar profile
- Many of them suffer from strong coupling problem, which is solved by scordatura (= controlled detuning of degeneracy condition)
- DHOST/Horndeski do not include scordatura but U-DHOST does (DeFelice, Mukohyama, Takahashi 2022) .
- EFT of ghost condensation already includes scordatura.
  Arkani-Hamed, Cheng, Luty and Mukohyama 2004

- Approximately stealth solution in scordatura is stealth at astrophysical scales and is free from the strong coupling problem
- Scordatura also helps recover the generalized second law

EFT of scalar-tensor gravity
with timelike scalar profile

EFT ON ARBITRARY BACKGROUND

arXiv: 2204.00228 w/ V.Yingcharoenrat

- Cosmology and black holes (BHs) play as important roles in gravitational physics as blackbody radiation and hydrogen atoms did in quantum mechanics.
- In cosmology a time-dependent scalar field can act as dark energy (DE), while BHs serve as probes of strong gravity. We then hope to probe the scalar field DE by astrophysical BHs.
- This would require the scalar field profile to be timelike near BH. Otherwise, contours of the scalar field would become ill-defined.
- Are there BH solution with timelike scalar profile?

- Cosmology and black holes (BHs) play as important roles in gravitational physics as blackbody radiation and hydrogen atoms did in quantum mechanics.
- In cosmology a time-dependent scalar field can act as dark energy (DE), while BHs serve as probes of strong gravity. We then hope to probe the scalar field DE by astrophysical BHs.
- This would require the scalar field profile to be timelike near BH. Otherwise, contours of the scalar field would become ill-defined.
- Are there BH solution with timelike scalar profile?
- Can we develop EFT of scalar-tensor gravity on such BH background?

It is not straightforward…

- General action in the unitary gauge ($\phi = \tau$)

$$S = \int d^4x \sqrt{-g}\, F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$$

- Taylor expansion around the background

$$S = \int d^4x \sqrt{-g} \left[\bar{F} + \bar{F}_{g^{\tau\tau}} \delta g^{\tau\tau} + \bar{F}_K \delta K + \cdots \right]$$

- The whole action is invariant under 3d diffeo but **each term is not…**
- Each coefficient is a function of (τ, x^i) but cannot be promoted to an arbitrary function.

Solution: consistency relations

- The chain rule

$$\begin{cases} \dfrac{d}{dx^i}\bar{F} = \bar{F}_{g^{\tau\tau}}\dfrac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K \dfrac{\partial \bar{K}}{\partial x^i} + \dots \\ \dfrac{d}{dx^i}\bar{F}_{g^{\tau\tau}} = \bar{F}_{g^{\tau\tau}g^{\tau\tau}}\dfrac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_{g^{\tau\tau}K} \dfrac{\partial \bar{K}}{\partial x^i} + \dots \\ \dfrac{d}{dx^i}\bar{F}_K = \bar{F}_{g^{\tau\tau}K}\dfrac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_{KK} \dfrac{\partial \bar{K}}{\partial x^i} + \dots \end{cases}$$

  relates x^i-derivatives of an EFT coefficient to other EFT coefficients, and leads to consistency relations.

- **The consistency relations ensure the spatial diffeo invariance.**
- Taylor coefficients should satisfy the consistency relations but are otherwise arbitrary.
- (No consistency relation for τ-derivatives.)

EFT action

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_\star^2}{2}f(y)R - \Lambda(y) - c(y)g^{\tau\tau} - \beta(y)K - \alpha^\mu_\nu(y)\sigma^\nu_\mu - \gamma^\mu_\nu(y)r^\nu_\mu + \frac{1}{2}m_2^4(y)(\delta g^{\tau\tau})^2$$
$$+\frac{1}{2}M_1^3(y)\delta g^{\tau\tau}\delta K + \frac{1}{2}M_2^2(y)\delta K^2 + \frac{1}{2}M_3^2(y)\delta K^\mu_\nu\delta K^\nu_\mu + \frac{1}{2}M_4(y)\delta K\delta^{(3)}R$$
$$+\frac{1}{2}M_5(y)\delta K^\mu_\nu\delta^{(3)}R^\nu_\mu + \frac{1}{2}\mu_1^2(y)\delta g^{\tau\tau}\delta^{(3)}R + \frac{1}{2}\mu_2(y)\delta^{(3)}R^2 + \frac{1}{2}\mu_3(y)\delta^{(3)}R^\mu_\nu\delta^{(3)}R^\nu_\mu$$
$$+\frac{1}{2}\lambda_1(y)^\nu_\mu\delta g^{\tau\tau}\delta K^\mu_\nu + \frac{1}{2}\lambda_2(y)^\nu_\mu\delta g^{\tau\tau}\delta^{(3)}R^\mu_\nu + \frac{1}{2}\lambda_3(y)^\nu_\mu\delta K\delta K^\mu_\nu + \frac{1}{2}\lambda_4(y)^\nu_\mu\delta K\delta^{(3)}R^\mu_\nu$$
$$+\frac{1}{2}\lambda_5(y)^\nu_\mu\delta^{(3)}R\delta K^\mu_\nu + \frac{1}{2}\lambda_6(y)^\nu_\mu\delta^{(3)}R\delta^{(3)}R^\mu_\nu + \dots\Big],$$

- EFT coefficients should satisfy the consistency relations but are otherwise arbitrary
- One can restore 4d diffeo by Stueckelberg trick
- Easy to find dictionary between EFT coefficients and theory parameters
- Can be applied to BH with timelike scalar profile
- Bridge between theories and observations

Applications to BHs with timelike scalar profile

- Background analysis for spherical BH
  [arXiv: 2204.00228 w/ V.Yingcharoenrat]
- Odd-parity perturbation around spherical BH
  → Generalized Regge-Wheeler equation
  [arXiv: 2208.02943 w/ K.Takahashi & V.Yingcharoenrat]
  [see also arXiv: 2208.02823 by Khoury, Noumi, Trodden, Wong]
  → Quasi-normal mode
  [work in progress w/ K.Takahashi & K.Tomikawa & V.Yingcharoenrat]
- Even-parity perturbation around spherical BH
  [work in progress w/ K.Takahashi & V.Yingcharoenrat]
- Rotating BH
- Dynamical BH
- …

EFT of scalar-tensor gravity
with timelike scalar profile

SUMMARY

- There are at least three motivations to go beyond GR:
(i) mysteries in the universe; (ii) quantum gravity;
(iii) understanding GR itself.
- Ghost condensation is a universal description of scalar-tensor gravity with timelike scalar profile on Minkowski background.
- Extension of ghost condensation to FLRW backgrounds results in a universal description of fluctuations of inflaton/dark energy (DE), called EFT of inflation/DE.
- If we want to learn something about scalar field DE from a black hole (BH) then we need to consider BH solutions with timelike scalar profile.

EFT of scalar-tensor gravity with timelike scalar profile

- **Time diffeo is broken by the scalar profile but spatial diffeo is preserved.**
- All terms that respect spatial diffeo must be included in the EFT action.
- Derivative & perturbative expansions
- Diffeo can be restored by introducing NG boson

EFT of scalar-tensor gravity on Minkowski background	= ghost condensation Arkani-Hamed, Cheng, Luty and Mukohyama, JHEP 0405:074,2004
EFT of scalar-tensor gravity on cosmological background	= EFT of inflation/dark energy Creminelli, Luty, Nicolis, Senatore 2006 Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore 2007
EFT of scalar-tensor gravity on arbitrary background	= EFT of BH perturbations arXiv: 2204.00228 w/ Vicharit Yingcharoenrat

Taylor expansion of the general action $S = \int d^4x \sqrt{-g}\, F(R_{\mu\nu\alpha\beta}, g^{\tau\tau}, K_{\mu\nu}, \nabla_\nu, \tau)$

$$S = \int d^4x \sqrt{-g} \left[\bar{F} + \bar{F}_{g^{\tau\tau}} \delta g^{\tau\tau} + \bar{F}_K \delta K + \quad \dots \right]$$

Consistency relations ⟵ S is invariant under spatial diffeo but the background breaks it.

$$\frac{d}{dx^i} \bar{F} = \bar{F}_{g^{\tau\tau}} \frac{\partial \bar{g}^{\tau\tau}}{\partial x^i} + \bar{F}_K \frac{\partial \bar{K}}{\partial x^i} + \dots$$

- There are at least three motivations to go beyond GR: (i) mysteries in the universe; (ii) quantum gravity; (iii) understanding GR itself.
- Ghost condensation is a universal description of scalar-tensor gravity with timelike scalar profile on Minkowski background.
- Extension of ghost condensation to FLRW backgrounds results in a universal description of fluctuations of inflaton/dark energy (DE), called EFT of inflation/DE.
- If we want to learn something about scalar field DE from a black hole (BH) then we need to consider BH solutions with timelike scalar profile.
- EFT of scalar-tensor gravity with timelike scalar profile on arbitrary background was developed. Consistency relations among EFT coefficients ensure the spatial diffeo invariance. Applicable to BHs with scalar field DE.
- Other applications?

Further extension of the web of EFTs

"The Effective Field Theory of Vector-Tensor Theories"
Katsuki Aoki, Mohammad Ali Gorji, Shinji Mukohyama, Kazufumi Takahashi, , JCAP 01 (2022) 01, 059 [arXiv: 2111.08119].

Residual symmetry in the unitary gauge

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

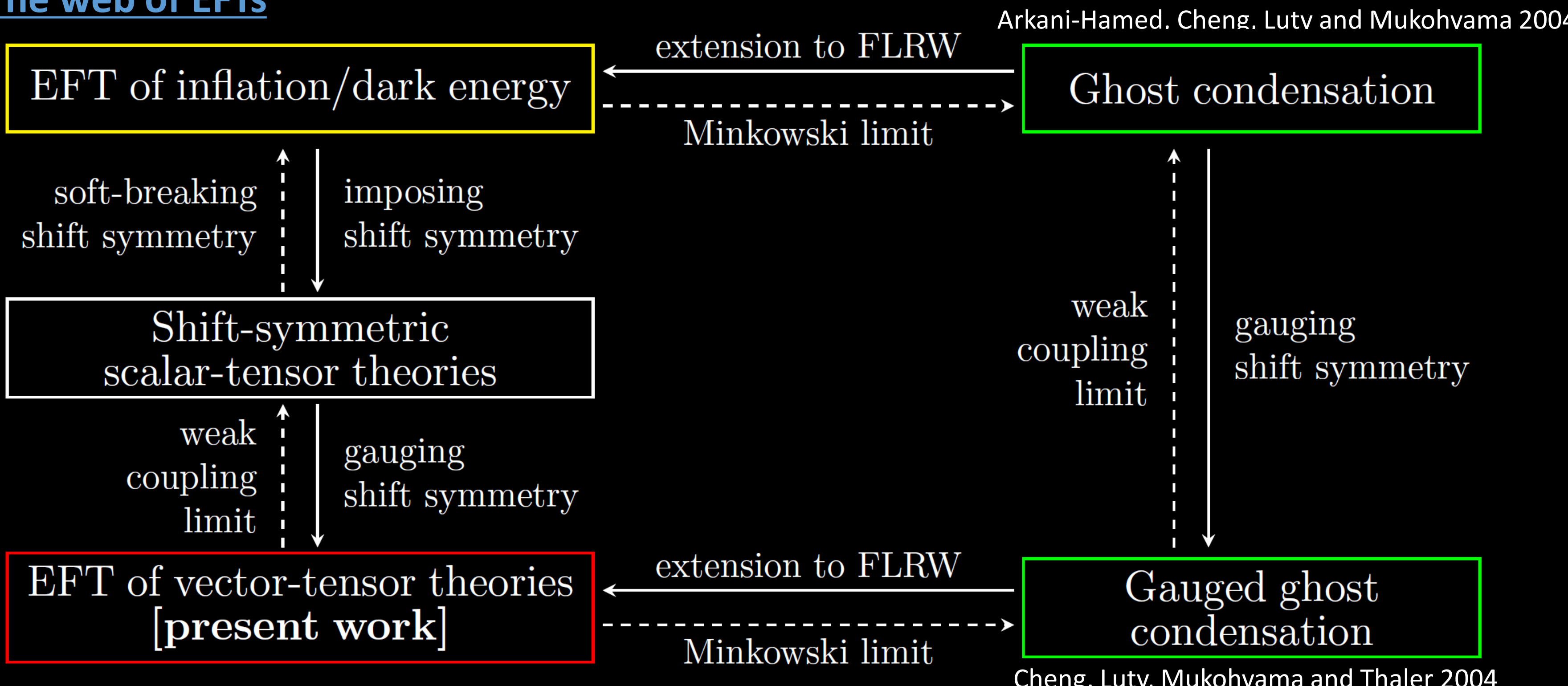
$$t \to t - g_M \chi(x)\,, \quad A_\mu \to A_\mu + \partial_\mu \chi(x)$$

leaving $\tilde{\delta}^0{}_\mu = \delta^0{}_\mu + g_M A_\mu$ invariant

c.f. Residual symmetry in unitary gauge for scalar-tensor theories

$$\vec{x} \to \vec{x}'(t, \vec{x})$$

The web of EFTs



Thank you!

K.Aoki

M.A.Gorji

K.Takahashi

V.Yingcharoenrat

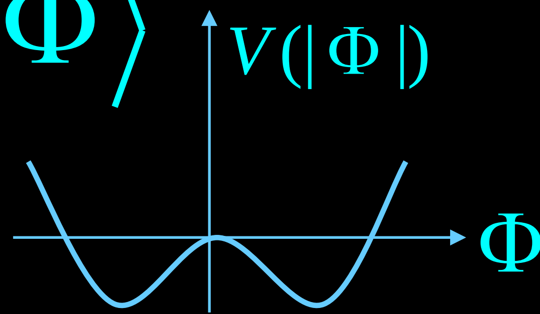
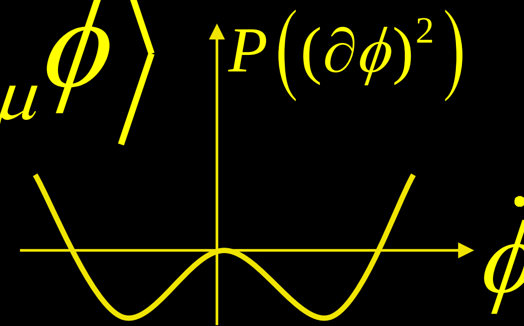
Ref. arXiv: 2204.00228 w/ V.Yingcharoenrat
arXiv: 2111.08119 w/ K.Aoki, M.A.Gorji & K.Takahashi

EFT of scalar-tensor gravity
with timelike scalar profile

Appendix A

GENERALIZED 2^{ND} LAW

Mukohyama 2009, 2010

	Higgs mechanism	***Ghost condensate*** Arkani-Hamed, Cheng, Luty and Mukohyama 2004
Order parameter	$\langle \Phi \rangle$ 	$\langle \partial_\mu \phi \rangle$ 
Instability	Tachyon $-\mu^2\Phi^2$	Ghost $-\dot{\phi}^2$
Condensate	V’=0, V’’>0	P’=0, P’’>0
Broken symmetry	Gauge symmetry	Time translational symmetry
Force to be modified	Gauge force	Gravity
New force law	Yukawa type	Newton+Oscillation

Bounds on symmetry breaking scale M

Arkani-Hamed, Cheng, Luty and Mukohyama and Wiseman 2007

0 100GeV 1TeV M

allowed

Jeans Instability (sun) — **ruled out**

Twinkling from Lensing (CMB) — **ruled out**

Supernova time-delay — **ruled out**

So far, there is no conflict with experiments and observations if M < 100GeV.

EFT of ghost condensation =

EFT of scalar-tensor gravity with timelike scalar profile on Minkowski background

Arkani-Hamed, Cheng, Luty and Mukohyama 2004

Backgrounds characterized by

- ✧ $\left\langle \partial_\mu \phi \right\rangle \neq 0$ and timelike
- ✧ Background metric is Minkowski.

⇨

$$\begin{aligned} L_{eff} = L_{EH} + M^4 \Big\{ & \left(h_{00} - 2\dot{\pi} \right)^2 - \frac{\alpha_1}{M^2} \left(K + \vec{\nabla}^2 \pi \right)^2 \\ & - \frac{\alpha_2}{M^2} \left(K^{ij} + \vec{\nabla}^i \vec{\nabla}^j \pi \right) \left(K_{ij} + \vec{\nabla}_i \vec{\nabla}_j \pi \right) + \cdots \Big\} \end{aligned}$$

Holography and GSL

- Do holographic dual descriptions always exist? PROBABLY NO. e.g.) A de Sitter space is only meta-stable and a unitary holographic dual is not known.
- How about ghost condensate?
- Let's look for violation of GSL in ghost condensate, since violation of GSL would indicate absence of holographic dual. (GSL is expected to be dual to ordinary 2^{nd} law.)
- Three proposals: (i) semi-classical heat flow; (ii) analogue of Penrose process; (iii) negative energy.

Different limits of speed

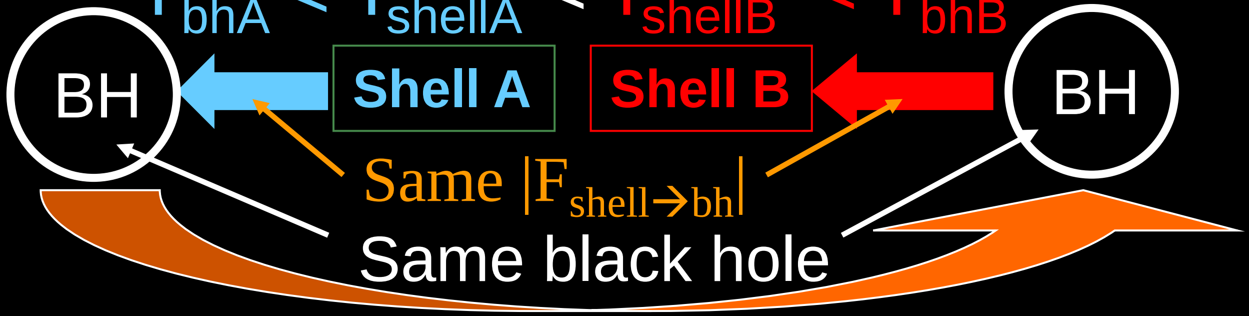
$$g_{A,B\mu\nu} = -u_\mu u_\nu + c_{A,B}^{-2}(g_{\mu\nu} + u_\mu u_\nu) \qquad u_\mu = \frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}}$$

- $\langle \partial_\mu \phi \rangle \sim M^2 \neq 0$ $\Rightarrow$ preferred direction u_μ.
- Different particles A and B may follow geodesics of different metrics $g_{A\mu\nu}$ and $g_{B\mu\nu}$.
- Lorentz breaking effects such as $|c_{A,B}{}^2-1|$ vanish in the limit $M^2 \rightarrow 0$ (M^2: order parameter)
  $c_{A,B}{}^2=1+O(M^2/M_{Pl}{}^2)$.

Semi-classical heat flow

Dubovsky and Sibiryakov 2006

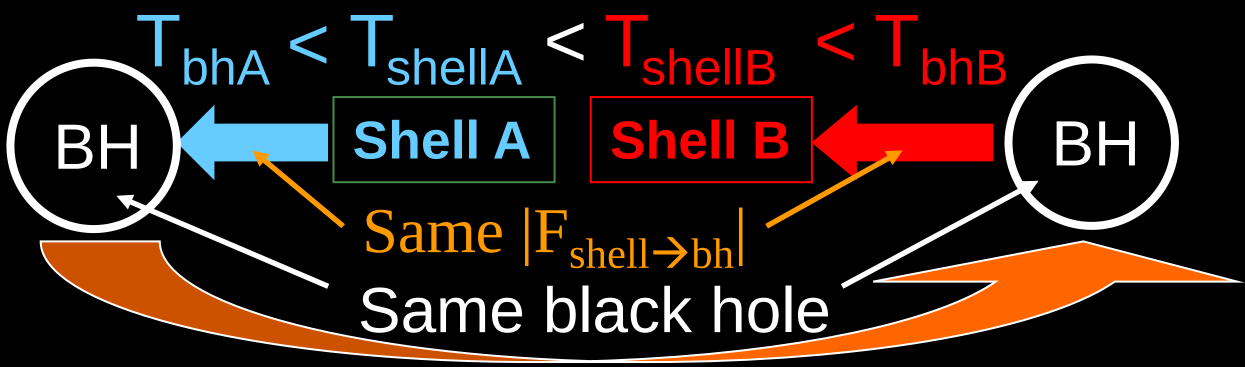
$T_{bhA} < T_{shellA} < T_{shellB} < T_{bhB}$



$dS_{shell}/dt =$
$(1/T_{shellB} - 1/T_{shellA})$
$*|F_{shell \rightarrow bh}| < 0$
$dS_{bh}/dt = 0$???

Semi-classical heat flow

Dubovsky and Sibiryakov 2006; Mukohyama 2009



$dS_{shell}/dt = (1/T_{shellB} - 1/T_{shellA}) * |F_{shell \to bh}| < 0$

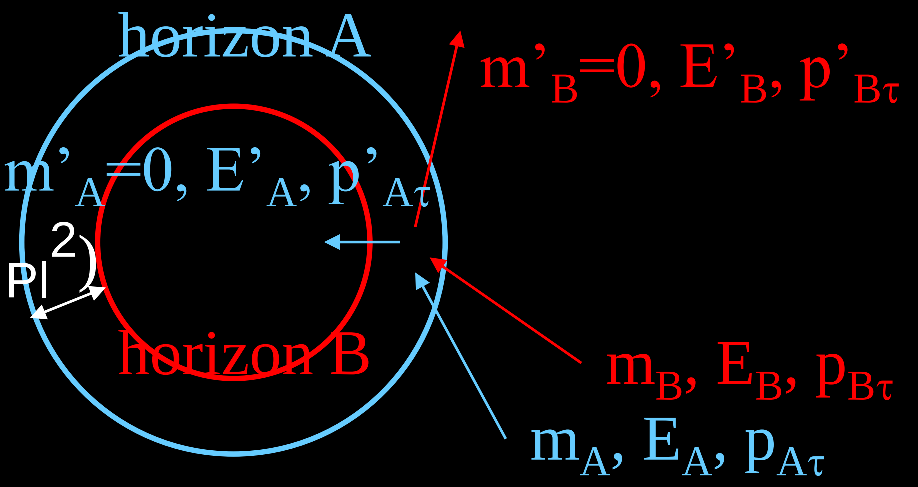
$dS_{bh}/dt = 0$???

GSL not violated!

- $|T_{bhB} - T_{bhA}| / T_{bh} = O(M^2/M_{Pl}^2)$
- $|F_{shell \to bh}| / T_{bh}^2 = O(M^2/M_{Pl}^2)$
- $|dS_{shell}/dt| / T_{bh} = O(M^4/M_{Pl}^4)$
- dS_{bh}/dt due to accretion is much larger.
- $S_{tot} = S_{shell} + S_{bh}$ does increase!

Analogue of Penrose process

Elling, Foster, Jacobson, Wall 2007



$\Delta r = r_g * O(M^2/M_{Pl}^2)$

$E'_A < 0$

$\Delta S_{bh} < 0$???

Analogue of Penrose process

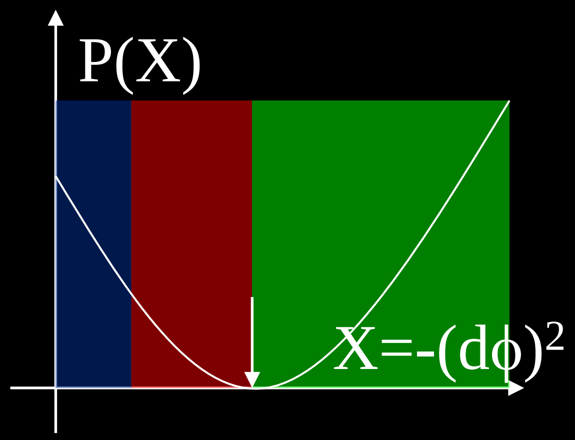
Elling, Foster, Jacobson, Wall 2007; Mukohyama 2010

GSL not violated!

- $c_A < c_B$ ➡ horizon A outside horizon B
- $E_A+E_B=E'_A+E'_B$, $p_{A\tau}+p_{B\tau}=p'_{A\tau}+p'_{B\tau}$
- Test particle approx. ➡ $m_{A,B}/M_{Pl}^2 << (r_g\Delta r)^{1/2}$
  ➡ $m_{A,B}^2/M_{bh}^2 << M^2/M_{Pl}^2$ ➡ $|E'_A|/M_{bh} << M^2/M_{Pl}^2$
- This process takes time scale ~ r_g , at least.
- $\Delta M_{bh,acc}/M_{bh} \sim M^2/M_{Pl}^2$ ➡ $\Delta S_{bh} > 0$!

Negative energy

Arkani-Hamed, talk at PI 2006



UV (ghost) instability

IR (gradient) instability

NEC violated

Ghost condensate

No instability

NEC OK

It appears that S_{bh} can be decreased by sending excitation with P’<0.

Averaged NEC

Mukohyama 2010

Action

$$I = \int dx^4 \sqrt{-g} P(X) \qquad X = -\partial^\mu \phi \partial_\mu \phi$$

Stress-energy tensor

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu} \qquad \rho = 2P' X - P \qquad u_\mu = \frac{\partial_\mu \phi}{\sqrt{X}}$$

EOM & shift charge

$$\nabla^\mu J_\mu = 0 \qquad J_\mu = -2P' \partial_\mu \phi \qquad Q = \int d\Sigma J_\mu u^\mu$$

In the regime of validity of EFT (|χ|<<1)

$$P = M^4 \left[p_0 + \tfrac{1}{2} p_2 \chi^2 + O(\chi^3) \right] \qquad \chi = \frac{X}{M^4} - 1$$

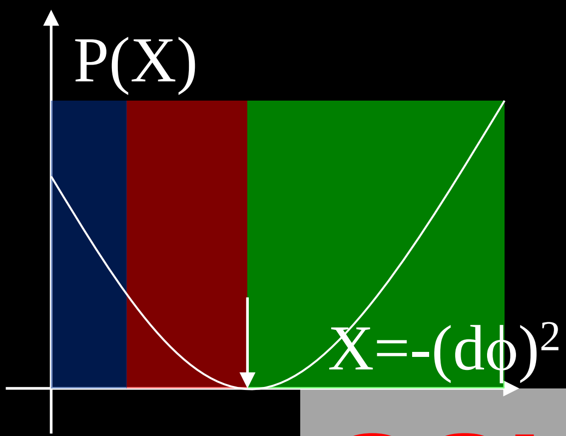
$$\rho + P - M^4 J_\mu u^\mu = M^4 \left[p_2 \chi^2 + O(\chi^3) \right]$$

Averaged NEC

$$\int d\Sigma (\rho + P) \geq M^2 Q \;\Rightarrow\; \int d\Sigma (\rho + P) \geq 0 \ \text{ for } \ Q \geq 0$$

Negative energy

Arkani-Hamed, talk at PI 2006; Mukohyama 2010



UV (ghost) instability

IR (gradient) instability

NEC violated

Ghost condensate

No instability

NEC OK

GSL not violated!

It appears that S_{bh} can be decreased by sending excitation with $P'<0$.

- GSL in a coarse-grained sense can be protected by the averaged NEC if the shift charge is non-negative. (Negative energy is followed by larger positive energy.)
- Negative charge states are plugged by instabilities in the early universe if the shift symmetry is exact. ($|P'|$ would be large in the early universe.)

EFT of scalar-tensor gravity
with timelike scalar profile

Appendix B

DE SITTER ENTROPY BOUND

Jazayeri, Mukohyama, Saitou, Watanabe 2016

Holography and GSL

- Do holographic dual descriptions always exist? PROBABLY NO. e.g.) A de Sitter space is only meta-stable and a unitary holographic dual is not known.
- How about ghost condensate?
- Let's look for violation of GSL in ghost condensate, since violation of GSL would indicate absence of holographic dual. (GSL is expected to be dual to ordinary 2^{nd} law.)
- Three proposals: (i) semi-classical heat flow; (ii) analogue of Penrose process; (iii) negative energy.
- **The generalized 2^{nd} law can hold in the presence of ghost condensate. (Mukohyama 2009, 2010)**

Ghost inflation and de Sitter entropy bound

Jazayeri, Mukohyama, Saitou, Watanabe 2016

- **Black holes & cosmology** in gravity theories are **as important as Hydrogen atoms & blackbody radiation** in quantum mechanics
- Provide **non-trivial tests** for theories of gravity e.g. black-hole entropy in string theory
- **Does the theory of ghost condensation pass those tests?**
- **Ghost condensation can be consistent with BH thermodynamics (Mukohyama 2009, 2010)**
- **How about de Sitter thermodynamics?**

de Sitter thermodynamics

- de Sitter (dS) spacetime is one of the three spacetimes with maximal symmetry
- dS horizon has temperature $T_H = H/(2\pi)$
- In quantum gravity, a dS space is probably unstable (e.g. KKLT, Susskind, …). So, let's consider a dS space as a part of inflation
- Friedmann equation →
  1st law with entropy $S = A/(4G_N) = \pi/(G_N H^2)$
  (This is in contrast with analogue gravity systems.)

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Slow roll inflation (non-eternal)

$$\dot{H} = -4\pi G_{\mathrm{N}} \dot{\phi}^2$$

$$S = \pi/(G_{\mathrm{N}} H^2) \qquad dN = H dt$$

$$\frac{\delta\rho}{\rho} \sim \frac{\delta a}{a} \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}} \sim \frac{H^2}{\dot{\phi}}$$

$$\frac{dS}{dN} = \frac{8\pi^2\dot{\phi}^2}{H^4} \sim \left(\frac{\delta\rho}{\rho}\right)^{-2}$$

$|\delta\rho/\rho| \lesssim 1$ for non-eternal inflation

$$\boxed{N_{\mathrm{tot}} \lesssim S_{\mathrm{end}} - S_{\mathrm{beginning}} < S_{\mathrm{end}}}$$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Eternal inflation

$$\delta\rho/\rho \gtrsim 1 \quad \Rightarrow \quad \Delta N \gtrsim \Delta S$$

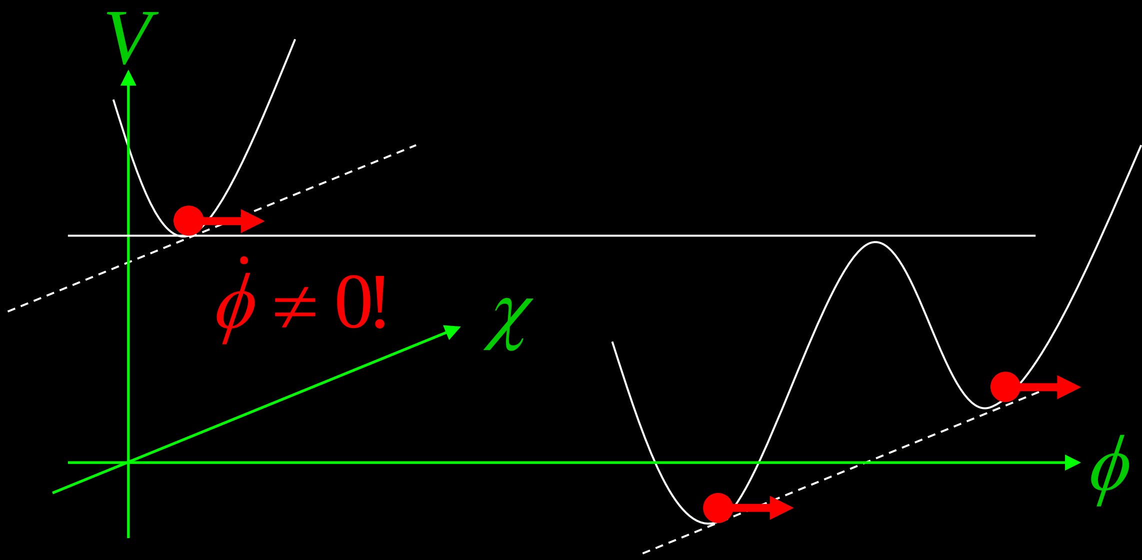
- Fluctuation generated during eternal epoch would collapse to form BH → unobservable!

$$\Rightarrow \quad \boxed{N_{\rm obs} \lesssim S_{\rm end}}$$

- This bound holds for a large class of models of inflation
- Does ghost inflation satisfy the bound?

Ghost inflation

Arkani-Hamed, Creminelli, Mukohyama and Zaldarriaga 2004



Similar to
hybrid inflation but
NOT SLOW ROLL

Scale-invariant perturbations

cf. tilted ghost inflation, Senatore (2004)

scaling dim of π

$$\frac{\delta\rho}{\rho} \sim \frac{H\delta\pi}{\dot{\phi}} \sim \left(\frac{H}{M}\right)^{5/4}$$

$$\delta\pi \sim M \cdot (H/M)^{1/4} \qquad \dot{\phi} \sim M^2$$

[compare $\frac{H}{M_{Pl}\sqrt{\varepsilon}}$]

Prediction of Large non-Gauss.

Leading non-linear interaction $\beta \dfrac{\dot{\pi}(\nabla\pi)^2}{M^2}$

non-G of $\sim \beta\left(\dfrac{H}{M}\right)^{1/4}$ ← scaling dim of op.

$\sim \beta\left(\dfrac{\delta\rho}{\rho}\right)^{1/5}$

$$\int dt d^3x\left[\frac{1}{2}\dot{\pi}^2 - \frac{\alpha(\vec{\nabla}^2\pi)^2}{M^2} + \cdots\right]$$

[Really "0.1" $\times(\delta\rho/\rho)^{1/5}$ ~ 10^{-2}. VISIBLE.
In usual inflation, non-G ~ $(\delta\rho/\rho)$ ~ 10^{-5} too small.]

⇒ $f_{NL} \sim 82\ \beta\alpha^{-4/5}$, equilateral type

Planck 2015 constraint (equilateral type)

$f_{NL} = -4 \pm 43$ (68% CL statistical) → $-0.6 \le \beta\alpha^{-4/5} \le 0.5$

de Sitter entropy bound

Arkani-Hamed, et.al. 2007

- Eternal inflation

$$\delta\rho/\rho \gtrsim 1 \quad \Rightarrow \quad \Delta N \gtrsim \Delta S$$

- Fluctuation generated during eternal epoch would collapse to form BH → unobservable!

$$\Rightarrow \quad \boxed{N_{\rm obs} \lesssim S_{\rm end}}$$

- This bound holds for a large class of models of inflation

- Does ghost inflation satisfy the bound?
The answer appears to be "no" since N_{tot} can be arbitrarily large. Swampland?

Lower bound on Λ?

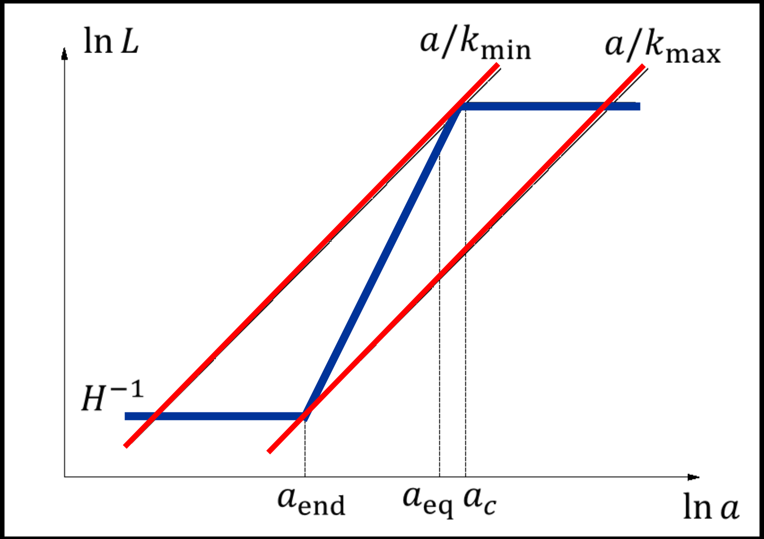
Jazayeri, Mukohyama, Saitou, Watanabe 2016

- Tiny Λ prevents earlier inflationary modes from being observed.

$$\frac{a_{\rm end}}{a_{\rm reh}} \sim \left(\frac{\rho_{\rm reh}}{\rho_{\rm inf}}\right)^{1/3} \qquad \frac{a_{\rm reh}}{a_{\rm eq}} \sim \left(\frac{s_{\rm eq}}{s_{\rm reh}}\right)^{1/3}$$

$\ddot{a}(t=t_c)=0$ with

$$6M_{\rm Pl}^2\frac{\ddot{a}}{a} = -\rho_{\rm m}^{\rm eq}\left(\frac{a_{\rm eq}}{a}\right)^3 + 2\rho_\Lambda$$



- $N_{\rm obs} \sim \ln(k_{\rm max}/k_{\rm min}) \lesssim S = \pi/(G_N H^2)$

$$\boxed{\Omega_\Lambda \gtrsim \exp\left[-10^{42}\left(\frac{M}{100\,{\rm GeV}}\right)^{-2}\right]} \qquad M \lesssim 100\,{\rm GeV}$$

- In our universe, Ω_Λ=O(1) and thus the bound is well satisfied.

Cosmological Page time

Jazayeri, Mukohyama, Saitou, Watanabe 2016

- Hawking rad from BH → $S_{rad} = S_{ent}$ increases but S_{BH} ($\geqq S_{ent}$) decreases → semi-classical description should break down @ Page time, when S_{BH} ~ half of $S_{BH,init}$
- After inflation, we expect to see O(1) deviation from semi-classical description @ Page time, when $N_{obs} \sim S_{end}$
- For example, if Λ decays at $a=a_{decay}$ then

$$\frac{a_{\rm Page}}{a_{\rm decay}} \sim \left(\frac{M}{100\,{\rm GeV}}\right)^{-1}\left(\frac{a_{\rm decay}}{a_{\rm eq}}\right)^{2}\exp\left[10^{42}\left(\frac{M}{100\,{\rm GeV}}\right)^{-2}\right]$$

Summary of appendices

- Ghost condensation is the simplest Higgs phase of gravity.
- The low-E EFT is determined by the symmetry breaking pattern. No ghost in the EFT.
- Gravity is modified in IR.
- Consistent with experiments and observations if $M < 100\mathrm{GeV}$.
- It appears easy but is actually difficult to violate the generalized 2nd law by ghost condensate.
- Ghost inflation predicts large non-Gaussianity that can be tested.
- de Sitter entropy bound appears to be violated but is actually satisfied by ghost inflation.