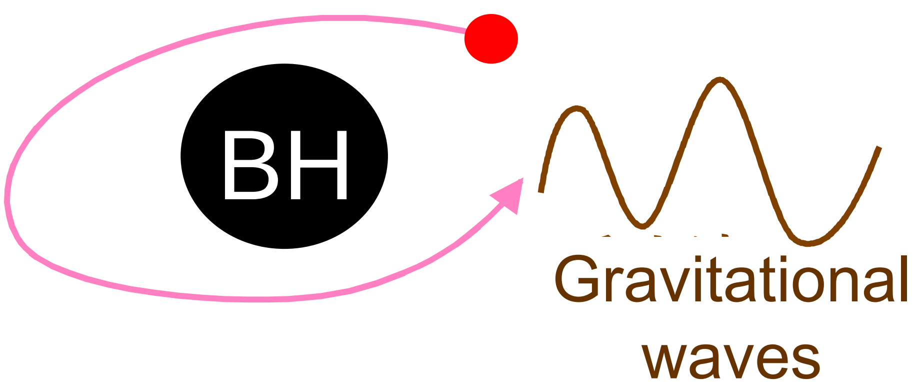
修正重力と重力波



田中 貴浩
(京大理/基研)

Motivation for modefied gravity

1) Incompleteness of General relativity

GR is non-renormalizabile

Singularity formation after gravitational collapse

$\Rightarrow$ Modification only at the Planck scale?

There are possibilities of modification even for the stellar mass BH from the holographic point of view.

2) Dark energy problem

Difficult to explain the smallness of dark energy, but anthropic argument may help.

"If the vacuum energy were slightly larger than the observed value, the universe would have started accelerated expansion before structure formation"

Nevertheless, it is interesting to seek the deviation from GR since it is getting possible to discriminate different models observationally.

Motivation for modefied gravity

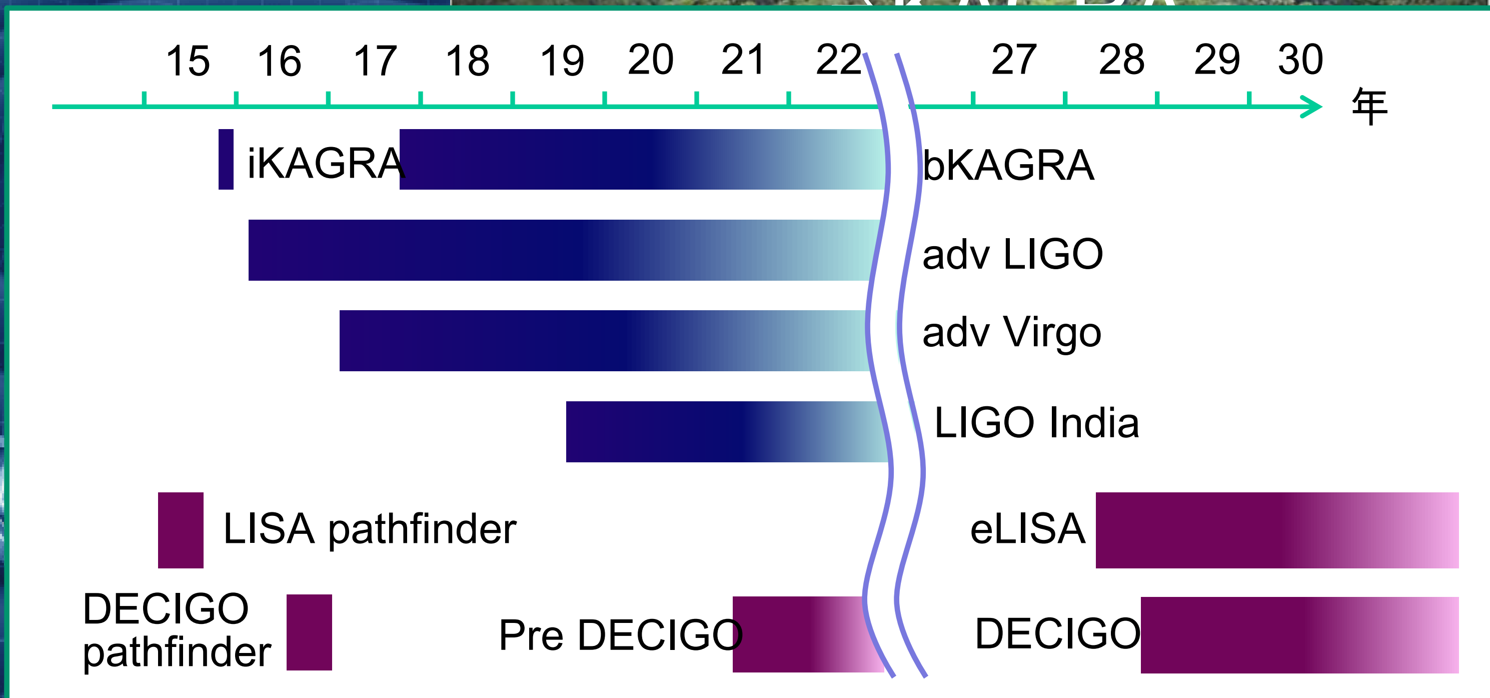
3) As an anti-these to General relativity

GR has been repeatedly tested since its first proposal.
The precision of the test is getting higher and higher.

⇒ Do we need to understand what kind of modification is theoretically possible before experimental test?

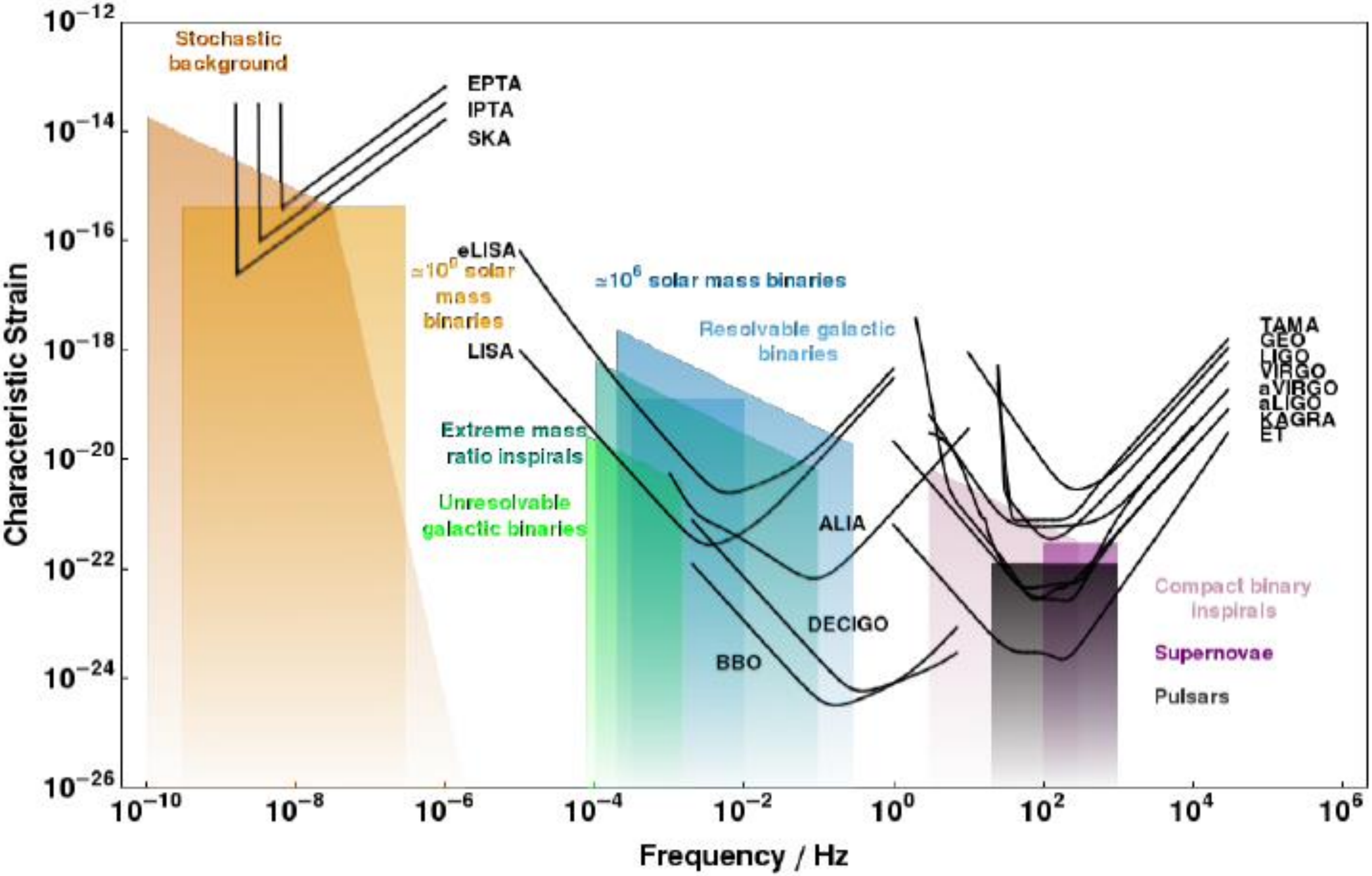
Yes, especially in the era of gravitational wave observation!

Gravitation wave detectors



eLISA(NGO)
⇒DECIGO/BBO

LIGO⇒adv LIGO



(Moore, Cole, Berry
http://www.ast.cam.ac.uk/~rhc26/sources/)

Inspiraling-coalescing binaries

- Various information from inspiral signal
  - Event rate
  - Binary parameters
  - EOS of nuclear matter
  - Test of GR

- Stellar mass BH/NS
  - Target of ground based detectors
  - Possible correlation with short γ-ray burst
  - primordial BH binaries (BHMACHO)
- Massive/intermediate mass BH binaries
  - Formation history of central super massive BHs
- Extreme (intermidiate) mass-ratio inspirals (EMRI)
  - Probe of BH geometry

- Inspiral phase (large separation)

  Clean system: ~ point particles (Cutler et al, PRL **70** 2984(1993))

  Internal structure of stars is not so important

  Accurate theoretical prediction of waveform is possible.

  - for detection
  - for parameter extraction(direction, mass, spin,…)
  - for precision test of general relativity

- Merging phase

  Numerical relativity

  - EOS of nuclear matter
  - Electromagnetic counterpart

- Ringing tail - quasi-normal oscillation of BH

Prediction of the event rate for binary NS mergers

Binary Systems Containing Radio Pulsars That Coalesce in Less than 10^{10} yr

PSR	P (ms)	P_b (hr)	e	Total Mass ($M_\odot$)	τ_c (Myr)	τ_{GW} (Myr)	Reference
J0737−3039A	22.70	2.45	0.088	2.58	210	87	Burgay et al. 2003
J0737−3039B	2773	2.45	0.088	2.58	50	87	Lyne et al. 2004
B1534+12	37.90	10.10	0.274	2.75	248	2690	Wolszczan 1990
J1756−2251	28.46	7.67	0.181	2.57	444	1690	This Letter
B1913+16	59.03	7.75	0.617	2.83	108	310	Hulse & Taylor 1975
B2127+11C	30.53	8.04	0.681	2.71	969	220	Anderson et al. 1990
J1141−6545†	393.90	4.74	0.172	2.30	1.4	590	Kaspi et al. 2000

} double pulsar

← NS-WD

Notes.—One NS-WD (†) and five DNS systems. PSR B2127+11C is in a globular cluster, implying a different formation history to the Galactic DNS systems. Here τ_c is the pulsars' characteristic age and τ_{GW} is the time remaining to coalesce due to emission of gravitational radiation. The total coalescence time is $\tau_c + \tau_{GW}$.

(Faulkner et al ApJ 618 L119 (2005))

total coalescence time

$$\tau(i) = \tau_c + \tau_{\mathrm{GW}}$$

Time to spin-down to the current spin velocity
+ time to elapse before coalescence

event rate per Milky way galaxy

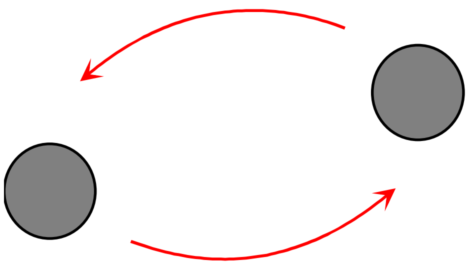
$$R = \sum_i \frac{V_{gal}}{V_{\max}(i)\,\tau(i)}$$

$V_{\max}(i)$ the volume in which we can detect an observed binary NS when it is placed there.

0.4 ~ 400yr^{-1} for advLIGO/Virgo (Abadie et al. 2010)

If short γ-ray bursts are binary NS mergers,
>1.5yr^{-1} for advanced detector network (Yonetoku et al. 1402.5463)

Theoretical prediction of GW waveform



Standard post Newtonian approximation
~ (v/c)expansion

4PN=$(v/c)^8$ computation is ready
(Blanchet, Living Rev.Rel.17:2
Damour et al. Phys.Rev. D89 (2014) 064058)

Waveform in Fourier space
for quasi-circular inspiral

$$h(f) \approx A f^{-7/6} e^{i\Psi(f)}$$

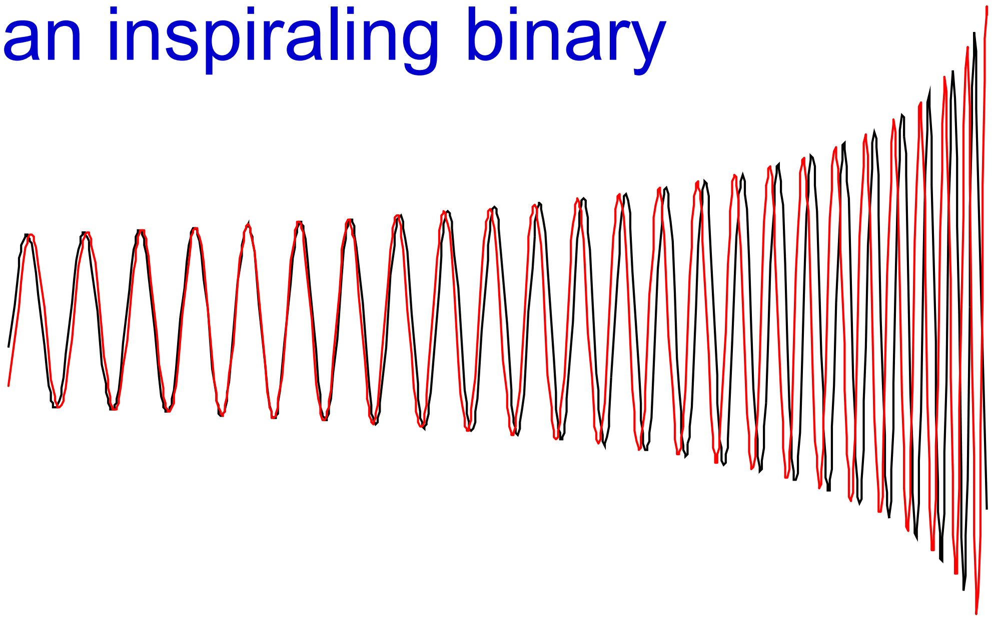
$$A = \frac{1}{\sqrt{20\pi^3}} \frac{\mathcal{M}^{5/6}}{D_L}, \quad \mathcal{M} = \mu^{3/5} M^{2/5}, \quad \eta = \frac{\mu}{M}$$

$$\Psi = 2\pi f t_c - \phi_c + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left[1 + \underbrace{\frac{20}{9}\left(\frac{743}{331} + \frac{11}{4}\eta\right)u^{2/3}}_{\text{1PN}} \underbrace{-(16\pi - \beta)u}_{\text{1.5PN}} + \cdots\right]$$

$$u \equiv \pi M f = O(v^3)$$

GR is correct in strong gravity regime?

Many cycles of gravitational waves from an inspiraling binary



1 cycle phase difference is detectable

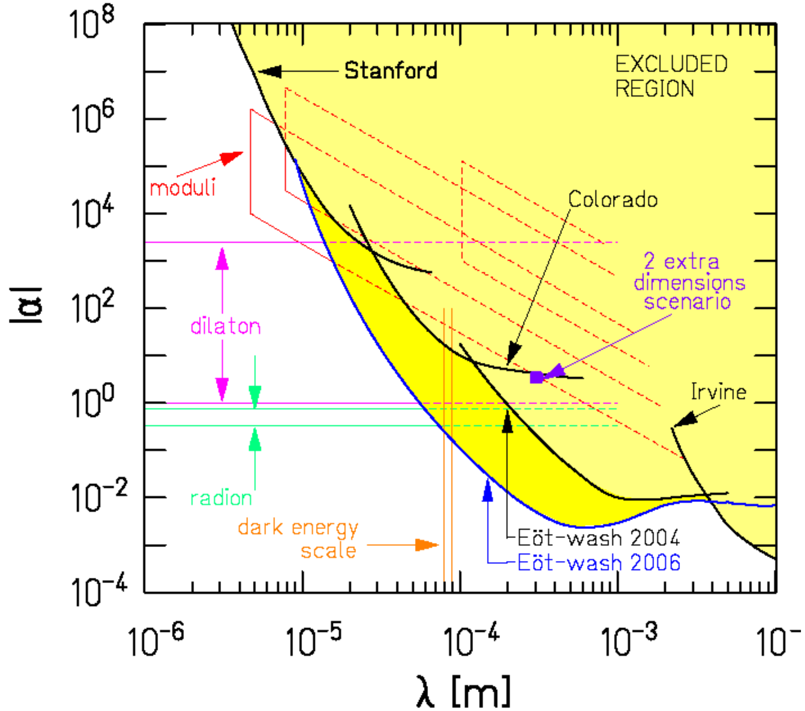
- Precise determination of orbital parameters
- Mapping of the strong gravity region of BH spacetime

Observational constraints on modified gravity theory

Deviation from the Newton's law

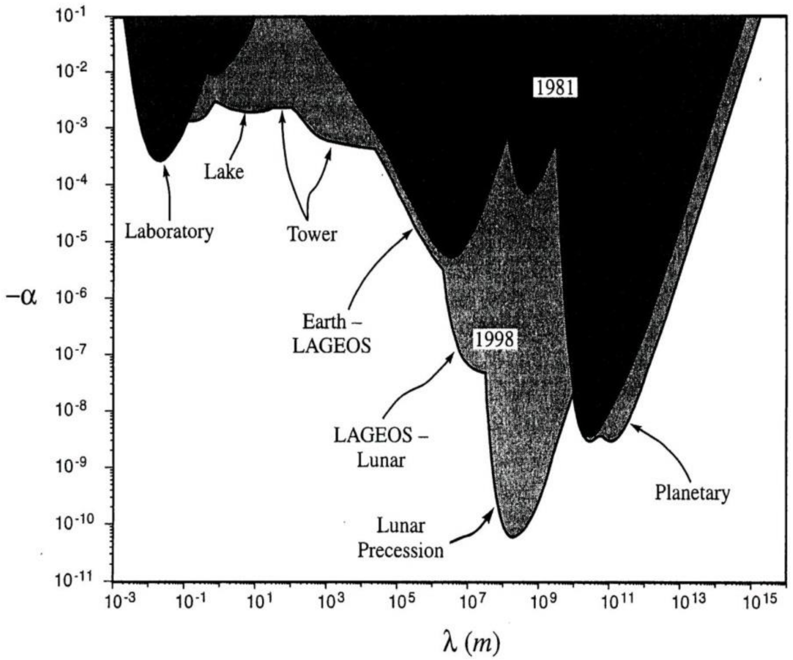
$$U = -\frac{G m_1 m_2}{r_{12}}\left[1 + \alpha \exp\left(-r_{12}/\lambda\right)\right]$$

Short range ~ sub-mm



(Capner et al, hep-ph/0611184)

Middle range ~ sub-AU



Fischbach & Talmadge

"The Search for Non-ewtonian Gravity" (1998)

• Parameterized post-Newton bounds

Ref) Will Living Rev.Rel. 17 (2014) 4

- γ : g_{ij} components
  - light bending

$$\delta\theta = \frac{1}{2}(1+\gamma)\times 1.75''$$

VLBI unpublished?

$$(\gamma - 1) \approx 3.2\times 10^{-4}$$

  - Shapiro time delav

$(\gamma - 1) \approx$ 2.3×10^{-5}

Cassini

$$1-\gamma = \frac{1}{2+\omega_{\mathrm{BD}}}$$

- β : 1PN g_{00} components

$$g_{00} = -1 + 2U - 2\beta U^2$$

  - perihelion shift

43 arcsec/100yr

$$\beta - 1 \approx 3\times 10^{-3}$$

  - Nordtvedt effect

Equivalence principle to the gravitational binding energy

$$a = \left(1 - 4(\beta - 1)\frac{E_g}{m}\right)\nabla U$$

$$\beta - 1 \approx 6\times 10^{-4}$$

Typical modification of GR

often discussed in the context of test by GWs

Scalar-tensor gravity

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R-\omega_{BD}\phi^{-1}\phi_{,\alpha}\phi^{,\alpha}\right)-\sum_a\int d\tau_a m_a(\phi)$$

$$G=\frac{4+2\omega_{BD}}{\phi(3+2\omega_{BD})}$$

scalar charge:

$$s_a=-\left[\partial(\ln m_a)/\partial(\ln G)\right]_0$$

G-dependence of the gravitational binding energy

$$\Psi=\cdots+\frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\left[\alpha u^{-2/3}+1+\left(\frac{3715}{756}+\frac{55}{9}\eta\right)u^{2/3}-(16\pi-\beta)u+\cdots\right]$$

Dipole radiation＝－1 PN frequency dependence

$$u=\pi Mf=O(v^3)$$

$$\alpha=-\frac{5(s_1-s_2)^2}{64\omega_{BD}}$$

For binaries composed of similar NSs, $(s_1-s_2)^2\ll 1$

Spontaneous scalarization

$$S=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R-\frac{\omega_{BD}(\phi)}{\phi}\phi_{,\alpha}\phi^{,\alpha}\right)$$

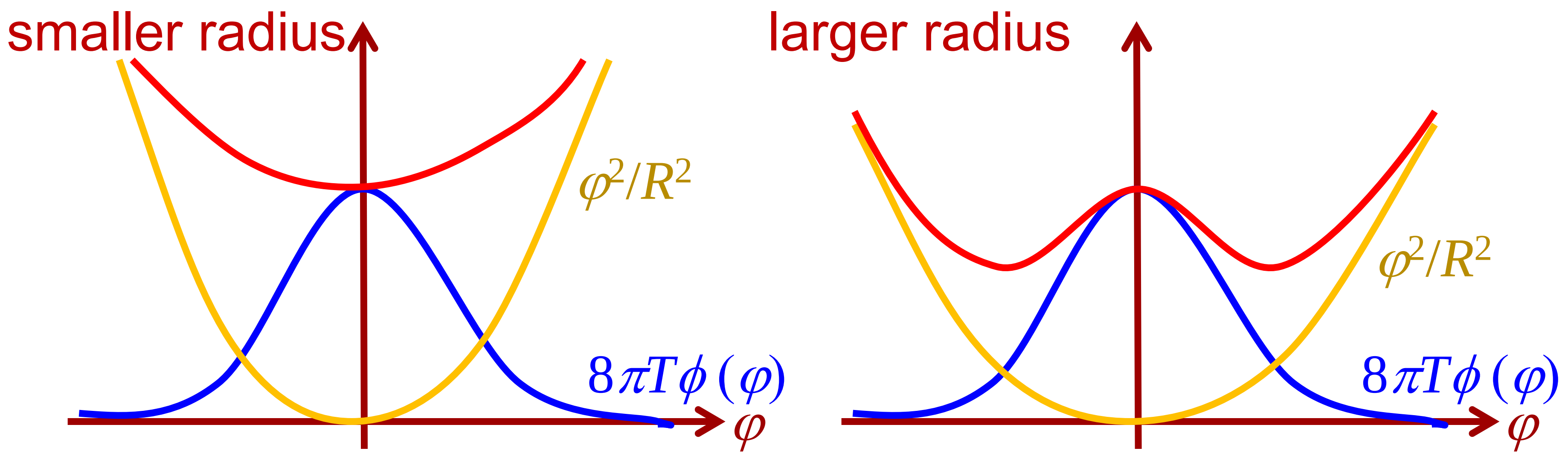
More general model

$$=\frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi(\varphi)R-\frac{1}{2}\varphi_{,\alpha}\varphi^{,\alpha}\right)$$

φ is canonically normalized

EOM $\Rightarrow$ $\Delta\varphi\approx-8\pi\phi'(\varphi)T$

Effective potential for a star with radius R.



As two NS get closer, "spontaneous scalarization" may happen.
Sudden change of structure and starting scalar wave emission.

Einstein Æther

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(R - M^{\alpha\beta}{}_{\mu\nu}\nabla_\alpha U^\mu \nabla_\beta U^\nu\right)$$

U is not coupled to matter field directly.

$$M^{\alpha\beta}{}_{\mu\nu} = c_1 g^{\alpha\beta} g_{\mu\nu} + c_2 \delta^\alpha_\mu \delta^\beta_\nu + c_3 \delta^\alpha_\nu \delta^\beta_\mu + c_4 U^\alpha U^\beta g_{\mu\nu}$$

with $U^\alpha U_\alpha = -1$

- At the lowest order in the weak field approximation, there is no correction to the metric if U^α // u^α (≡the four momentum of the star).
- The Lorentz violating effects should be suppressed.

⇨ two constraints among the four coefficients

Compact self-gravitating bodies can have significant scalar charge due to the strong gravity effect.

⇨ Dipole radiation.

Scalar-tensor gravity (conti)

Current constraint on dipole radiation:

$\omega_{BD} > 2.4 \times 10^4$ J1141-6545

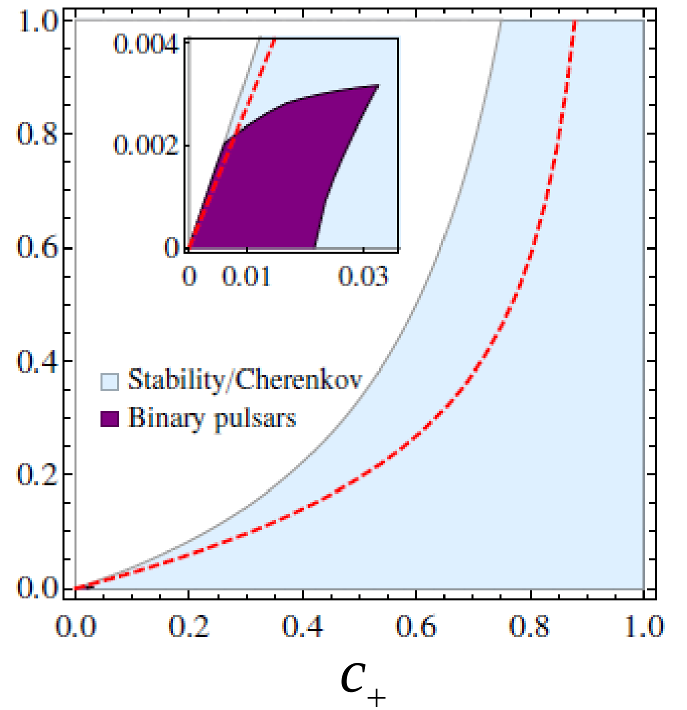
(NS(young pulsar)-WD)

(Bhat et al. arXiv:0804.0956)

The case of Einstein Æther ⇒

$$c_\pm = c_1 \pm c_3$$

(Yagi et al. arXiv:1311.7144)



Constraint from future observations:

(Yagi & TT, arXiv:0908.3283)

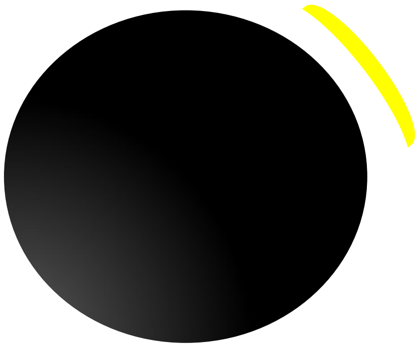
LISA– $1.4M_\odot$NS+$1000M_\odot$BH: $\omega_{BD} > 5 \times 10^3$

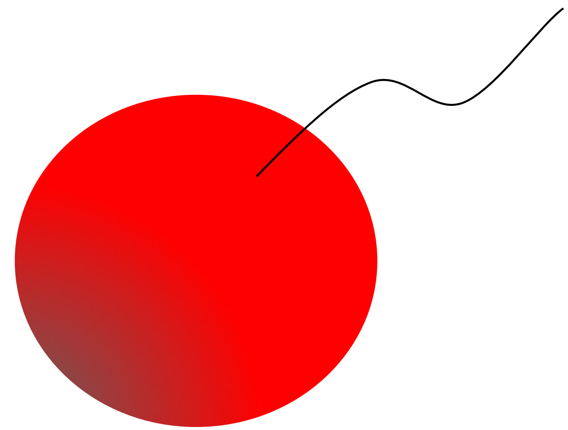
at 40Mpc corresponding to $SNR = \sqrt{200}$

Decigo–$1.4M_\odot$NS+$10M_\odot$BH: $\omega_{BD} > 8 \times 10^7$

collecting 10^4events at cosmological distances

Scalar-tensor theory
BH no hair





Einstein dilaton Gauss-Bonnet, Chern-Simons gravity

$$S \supset \frac{\alpha}{G_N}\int d^4x\sqrt{-g}\,\theta\begin{pmatrix} R_{GB} \\ {}^{*}R\,R \end{pmatrix} - \frac{1}{2G_N}\int d^4x\sqrt{-g}\left[(\partial\theta)^2 + 2V(\theta)\right]$$

$\theta\times$(higher curvature)

$$R_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R^{\alpha\beta}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta} \qquad {}^{*}R\,R = \varepsilon^{\alpha\beta}{}_{\sigma\chi}R^{\sigma\chi}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta}$$

- For constnat θ, these higher curvature terms are topological invariant. Hence, no effect on EOM.
- Higher derivative becomes effective only in strong field.

Hairy BH - bold NS

- NS in EDGB and CS do not have any scalar charge.

$$\Box\theta \approx "R^2" \Rightarrow Q = \int d^3x\, "R^2" = \frac{1}{T}\int \underline{d^4x\, "R^2"}$$

topological invariant, which vanishes on topologically trivial spacetime.

- By contrast, BH solutions in EDGB and CS have scalar monopole and dipole, respectively.

EDGB : monopole charge ⇒ dipole radiation (-1PN order)

CS : dipole charge ⇒ 2PN order

(Yagi, Stein, Yunes, Tanaka (2012))

Observational bounds

- EDGB

Cassini $\alpha_{EDGB}^{1/2} < 1.3 \times 10^{12}\,\mathrm{cm}$ (Amendola, Charmousis, Davis (2007))

Low mass X-ray binary, A0620-00

$$\alpha_{EDGB}^{1/2} < 1.9 \times 10^{5}\,\mathrm{cm}$$ (Yagi, arXiv:1204.4525)

Future Ground-based GW observation

SNR=20, 6Msol + 12Msol

$$\alpha_{EDGB}^{1/2} < 4 \times 10^{5}\,\mathrm{cm}$$ (Yagi, Stein, Yunes, TT, arXiv:1110.5950)

- CS

Gravity Probe B, LAGEOS (Ali-Haimound, Chen (2011))

$$\alpha_{CS}^{1/2} < 10^{13}\,\mathrm{cm}$$

Future Ground-based GW observation with favorable spin alignment: 100Mpc, $a \sim 0.4$M

$$\alpha_{CS}^{1/2} < 10^{6-7}\,\mathrm{cm}$$ (Yagi, Yunes, TT, arXiv:1208.5102)

Simple addition of mass to graviton

phase velocity of massive graviton

$$c_{phase}(f) = \frac{k}{\omega} \approx 1 - \frac{m^2}{2\omega^2} = 1 - \frac{1}{2\lambda_g^2 f^2}$$

$$D = \int d\eta\, a^2$$

$$\Rightarrow \Delta\Psi = 2\pi f \Delta t = 2\pi f D \Delta c_{phase}(f) \approx -\frac{\pi D}{\lambda_g^2 f}$$

Phase shift depending on frequencies

$$\Psi = \cdots + \frac{3}{128}(\pi \mathcal{M} f)^{-5/3}\left[1 + \left(\frac{3715}{756} + \frac{55}{9}\eta - \frac{128}{3}\eta\beta_g\right)u^{2/3} - (16\pi - \beta)u + \cdots\right]$$

$$u = \pi M f = O(v^3)$$

Graviton mass effect

$$\beta_g = \frac{\pi^2 D M}{\lambda_g^2}$$

Constraint from future observations:

LISA– $10^7 M_\odot$BH+$10^6 M_\odot$BH at3Gpc:

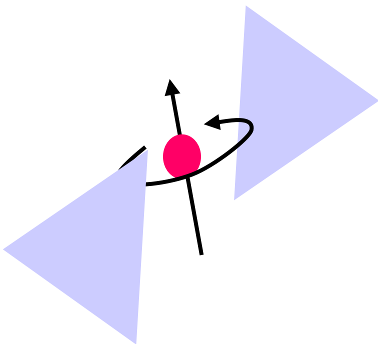
graviton compton wavelength

$\lambda_g > 4\text{kpc}$

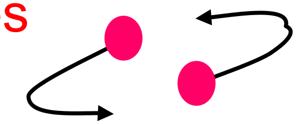
(Yagi & TT, arXiv:0908.3283)

Test of GW generation

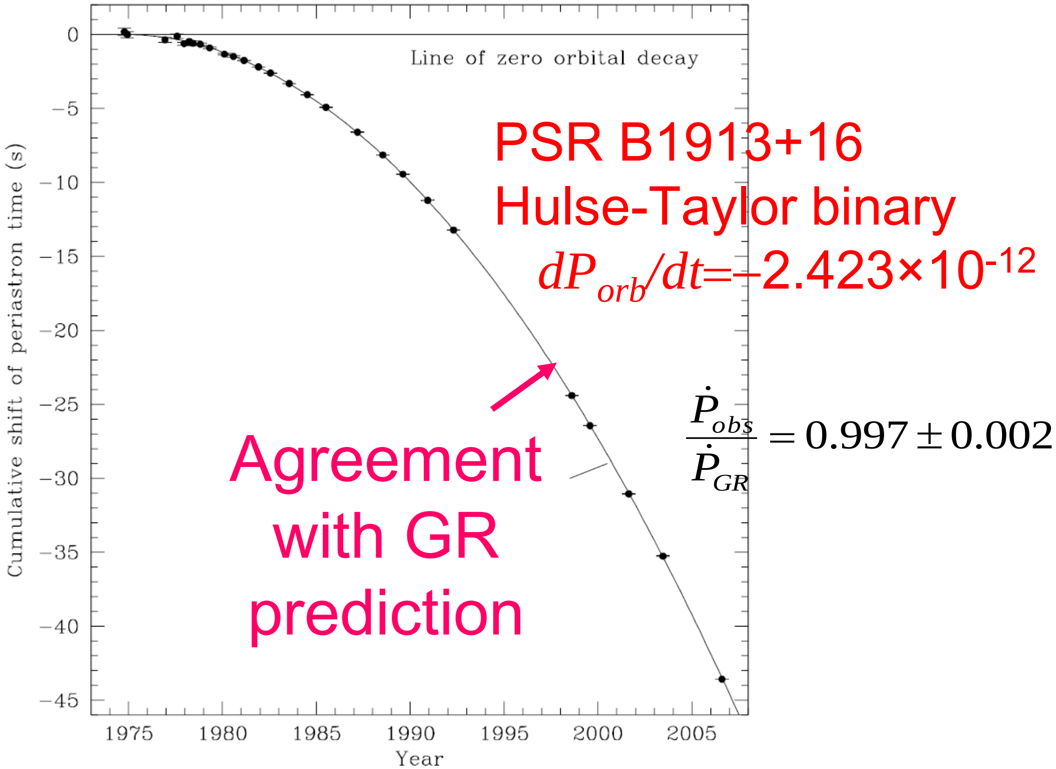
Pulsar : ideal clock



Test of GR by pulsar binaries



Periastron advance due to GW emission



(J.M. Weisberg, Nice and J.H. Taylor, arXiv:1011.0718)

We know that GWs are emitted from binaries.

But, then
what can be a big surprise
when we first detect GWs?

Is there any possibility that
gravitons disappear during its propagation
over a cosmological distance?

Test of gravitational wave propagation

Just fast propagation of GWs can be realized in Einstein Æther model.

But how can we realize the famous Nakamura-san's first dream of the New Year?

20??年、10年間のデータを解析した研究チームは次のような結果を得て、記者会見をした。「PSR1913+16までの距離は重力波によると4万光年で、電波天文学による距離2万光年をどのように見積もっても約2倍大きい。つまり、重力波が伝播している間に1部が消えていると考えざるを得ない…」

Chern-Simons Modified Gravity

$$S \supset \frac{\alpha}{8}\int d^4x\sqrt{-g}\,\theta\,\varepsilon^{\alpha\beta}{}_{\sigma\chi}R^{\sigma\chi}{}_{\mu\nu}R^{\mu\nu}{}_{\alpha\beta} - \frac{\beta}{2}\int d^4x\sqrt{-g}\left[(\partial\theta)^2 + 2V(\theta)\right]$$

Right-handed and left-handed gravitational waves are amplified/decreased differently during propagation, depending on the frequencies. (Yunes & Spergel, arXiv:0810.5541)

$$\mathbf{h}^{(L,R)} = \frac{1}{\sqrt{2}}\left(\mathbf{h}^{(+)} + i\mathbf{h}^{(\times)}\right)$$

The origin of this effect is clear in the effective action.

(Flanagan & Kamionkowski, arXiv:1208.4871)

$$S^{(2)} = \frac{m_p^2}{4}\int d\eta\, d^3k \sum_{A=L,R} a^2(\eta)\left[1 + \lambda_A \omega \frac{\alpha\dot{\theta}}{M_p^2}\right]\left(\left|h_{A\boldsymbol{k},\eta}\right|^2 - k^2\left|h_{A\boldsymbol{k},\eta}\right|\right)$$

The time variation of this factor affects the amplitude of GWs.

$$S = \frac{m_p^2}{4}\int d\eta\, d^3k \sum_{A=L,R} a^2(\eta)\left[1+\lambda_A \omega \frac{\alpha\dot{\theta}}{M_p^2}\right]\left(\left|h_{A\boldsymbol{k},\eta}\right|^2 - k^2\left|h_{A\boldsymbol{k},\eta}\right|\right)$$

$$\boldsymbol{h}_{\mathrm{obs}}^{(L,R)} \approx \boldsymbol{h}_{\mathrm{emit}}^{(L,R)} \sqrt{1 \pm \frac{\omega\alpha\dot{\theta}}{M_p^2}}\Bigg|_{\mathrm{emit}} \Bigg/ \sqrt{1 \pm \frac{\omega\alpha\dot{\theta}}{M_p^2}}\Bigg|_{\mathrm{obs}}$$

Current constraint on the evolution of the background scalar field θ :

$$\left|\alpha\dot{\theta}\right| < \left(10^6\,\mathrm{Hz}\right)^{-1}$$

: J0737-3039(double pulsar)

(Ali-Haimoud, (2011))

But the model has a ghost for large ω, and the variation of GW amplitude is significant only for marginally large ω.

In other words, $\left|\omega\alpha\dot{\theta}\right| \approx 1$ modes are in the strong coupling regime, which is outside the validity of effective field theory.

Bi-gravity

(De Felice, Nakamura, TT arXiv:1304.3920)

Massive gravity

$$\Box h_{\mu\nu} = 0 \quad \Rightarrow \quad \left(\Box - m^2\right) h_{\mu\nu} = 0$$

Simple graviton mass term is theoretically inconsistent → ghost, instability, etc.

⇒ Bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{16\pi} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{16\pi\kappa} + \frac{L_{matter}(g,\phi)}{M_G^2} + \cdots$$

Both massive and massless gravitons exist.

→ ν oscillation-like phenomena?

First question is whether or not we can construct a viable cosmological model.

1) Ghost-free bigravity model exists.

2) It has a FLRW background very similar to the GR case at low energy.

3) The non-linear mechanism seems to work to pass the solar system constraints. (Vainshtein mechanism)

4) Two graviton eigen modes are superposition of two metric perturbations, which are mass eigen states at low frequencies and δg and $\delta\tilde{g}$ themselves at high frequencies.

5) Graviton oscillations occur only at around the crossover frequency, but there is some chance for observation.

Ghost free bi-gravity

$$\frac{L}{M_G^2} = \frac{\sqrt{-g}R}{2} + \frac{\sqrt{-\tilde{g}}\tilde{R}}{2\kappa} + \frac{\sqrt{-g}}{2}\sum_{n=0}^{4} c_n V_n + \frac{L_{matter}}{M_G^2}$$

$$V_0 = 1,\ V_1 = \tau_1, V_2 = \tau_1^2 - \tau_2,\ \cdots$$

$$\tau_n \equiv Tr\left[\gamma^n\right] \quad \gamma^i_j \equiv \sqrt{g^{ik}\tilde{g}_{kj}}$$

When $\tilde{g}$ is fixed, de Rham-Gabadadze-Tolley massive gravity.

Even if $\tilde{g}$ is promoted to a dynamical field, the model remains to be free from ghost.

(Hassan, Rosen (2012))

FLRW background

(Comelli, Crisostomi, Nesti, Pilo (2012))

Generic homogeneous isotropic metrics

$$ds^2 = \underline{a^2(t)}\left(-dt^2 + dx^2\right)$$
$$d\tilde{s}^2 = \underline{b^2(t)}\left(-\underline{c^2(t)}dt^2 + dx^2\right)$$

$$\xi \equiv b/a$$

$$\Rightarrow \underbrace{\left(6c_3\xi^2 + 4c_2\xi + c_1\right)}_{\text{branch 1}}\underbrace{\left(cba' - ab'\right)}_{\text{branch 2}} = 0$$

branch 1 : Pathological:

Strong coupling

Unstable for the homogeneous anisotropic mode.

branch 2 : Healthy

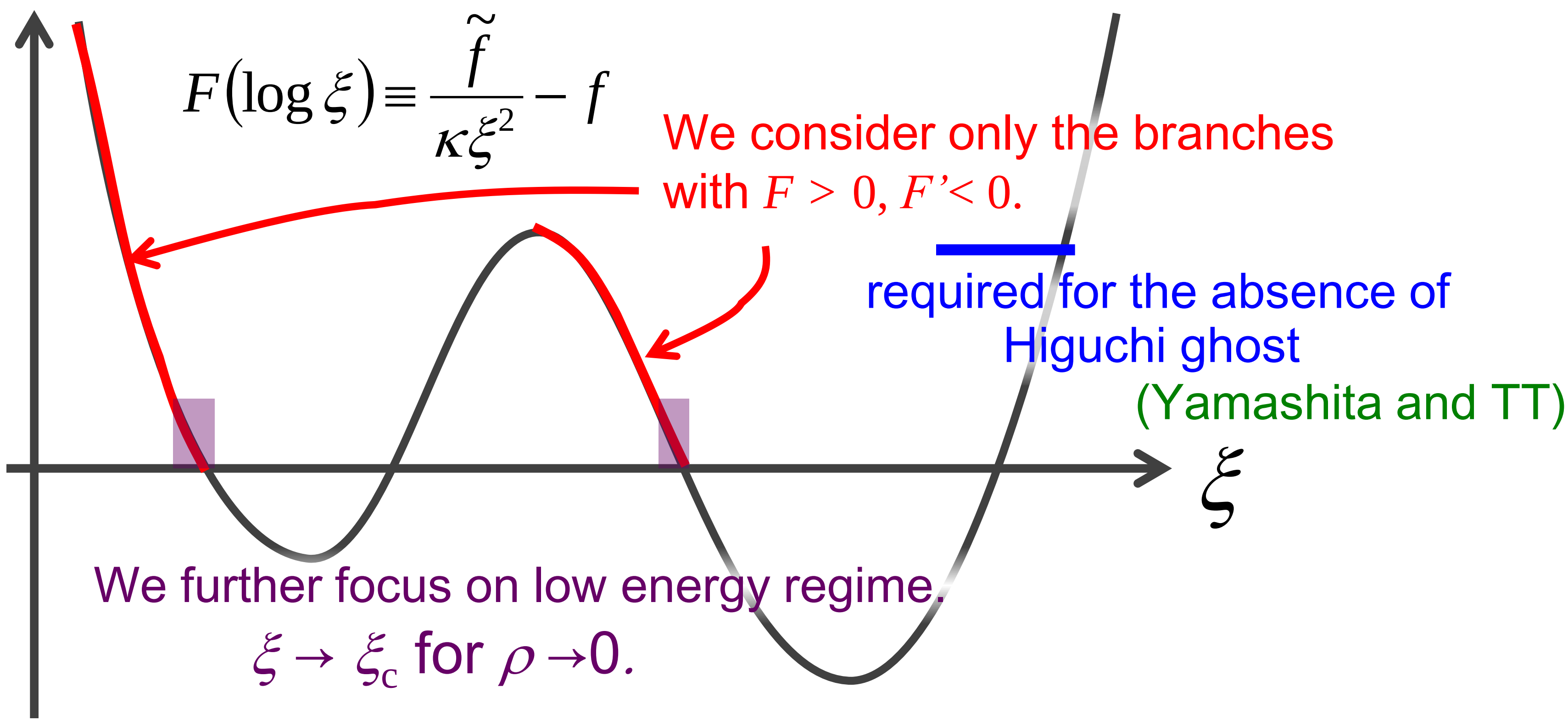
Branch 2 background

A very simple relation holds:

$$\frac{\rho}{M_G^2} + f - \tilde{f}/\kappa\xi^2 = 0 \qquad f(\log\xi) := c_0 + 3c_1\xi + 6c_2\xi^2 + 6c_3\xi^3$$

$$\tilde{f}(\log\xi) := c_1\xi + 6c_2\xi^2 + 18c_3\xi^3 + 24c_4\xi^4$$

$\xi \equiv b/a$ is algebraically determined as a function of ρ.



Branch 2 background

We expand with respect to $\delta\xi = \xi - \xi_c$.

$$H^2 = \frac{\rho}{3M_G^2} + \frac{f}{3} \Rightarrow H^2 = \frac{\rho}{3(1+\kappa\xi_c^2)M_G^2}$$

effective energy density due to mass term

Effective gravitational coupling is weaker because of the dilution to the hidden sector.

$$\frac{1}{c-1}\frac{\xi'}{\xi} = \frac{a'}{a} \Rightarrow c-1 = \frac{3(\rho+P)}{\mu^2 M_G^2}$$

Effective graviton mass

$$\mu^2 = \left(1 + \frac{1}{\kappa\xi_c^2}\right) f_c'$$

natural tuning to coincident light cones (c=1) at low energies ($\rho \to 0$)!

Solar system constraint: basics

- vDVZ discontinuity

In GR, this coefficient is 1/2

current bound <10^{-5}

$$\delta g_{\mu\nu} \propto \Box^{-1}\left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T\right)$$

To cure this discontinuity

we go beyond the linear perturbation (Vainshtein)

Schematically

Correction to the Newton potential Φ

$$\Delta\delta\Phi + \mu^{-2}\left(\partial\partial\delta\Phi\right)^2 = G_N \rho$$

$$\Rightarrow \frac{\delta\Phi}{\Phi} \approx \frac{\mu r^2 \sqrt{G_N \rho}}{r^2 G_N \rho} \approx \mu \sqrt{\frac{r^3}{r_g}}$$

$$10^{-10} \geq \mu \sqrt{\left(10^{13} cm\right)^3 / \left(10^5 cm\right)} \Rightarrow \mu^{-1} \geq 300 Mpc$$

Gravitational potential around a star in the limit $c \to 1$

Spherically symmetric static configuration:

$$ds^2 = -e^{u-v}dt^2 + e^{u+v}\left(dr^2 + r^2 d\Omega^2\right)$$

$$d\tilde{s}^2 = \xi_c^2\left[-e^{\tilde{u}-\tilde{v}}dt^2 + e^{\tilde{u}+\tilde{v}}\left(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2\right)\right] \qquad \tilde{r} = e^R r$$

Erasing $\tilde{u}, \tilde{v}$ and R ,

$$\Rightarrow \quad \left(\Delta - \mu^2\right)u - \frac{C}{\mu^2}\left((\Delta u)^2 - \left(\partial_i \partial_j u\right)^2\right) \approx \frac{\rho_m}{M_G^2}$$

$C \propto f_c''$, which can be tuned to be extremely large.

Then, the Vainshtein radius $r_V \approx \left(\dfrac{C r_g}{\mu^2}\right)^{1/3}$

can be made very large, even if $\mu^{-1} << 300\mathrm{Mpc}$.

Solar system constraint: $\sqrt{C}\,\mu^{-1} \geq 300\mathrm{Mpc}$

$$\Delta v \approx \frac{\rho_m}{\tilde{M}_G^2}$$

v is excited as in GR.

$$H^2 = \frac{\rho}{3\tilde{M}_G^2}$$

Excitation of the metric perturbation on the hidden sector:

Erasing u, v and R

$$\Rightarrow \left(\Delta-\mu^2\right)\tilde{u}-\frac{\tilde{C}}{\mu^2}\left(\left(\Delta\tilde{u}\right)^2-\left(\partial_i\partial_j\tilde{u}\right)^2\right)\approx\frac{\rho_m}{M_G^2}$$

$$\Delta\tilde{v}\approx\frac{\rho_m}{\tilde{M}_G^2}$$

$\tilde{u}$ is also suppressed like u.

$\tilde{v}$ is also excited like v.

The metric perturbations are almost conformally related with each other: $d\tilde{s}^2\approx\xi_c^2 ds^2$

Non-linear terms of $\tilde{u}$ (or equivalently u) play the role of the source of gravity.

Gravitational wave propagation

Short wavelength approximation :

$$k >> m_g >> H$$

$$h'' - \Delta h + m_g^2 \left(h - \tilde{h}\right) = 0$$

$$\tilde{h}'' - c^2 \Delta \tilde{h} - \frac{c m_g^2}{\kappa \xi_c^2} \left(h - \tilde{h}\right) = 0$$

$$m_g^2 = \frac{f'}{3} + \frac{(c-1)}{6}\left(f'' - f'\right)$$

(Comelli, Crisostomi, Pilo (2012))

$$\mu^2 := m_g^2 \frac{1+\kappa\xi^2}{\kappa\xi^2}$$

$$k_c := \frac{\mu}{\sqrt{2(c-1)}}$$

k

mass term is important.

Eigenmodes are

$$h + \tilde{h}, \quad \kappa \xi_c^2 h - \tilde{h}$$

modified dispersion relation due to the effect of mass

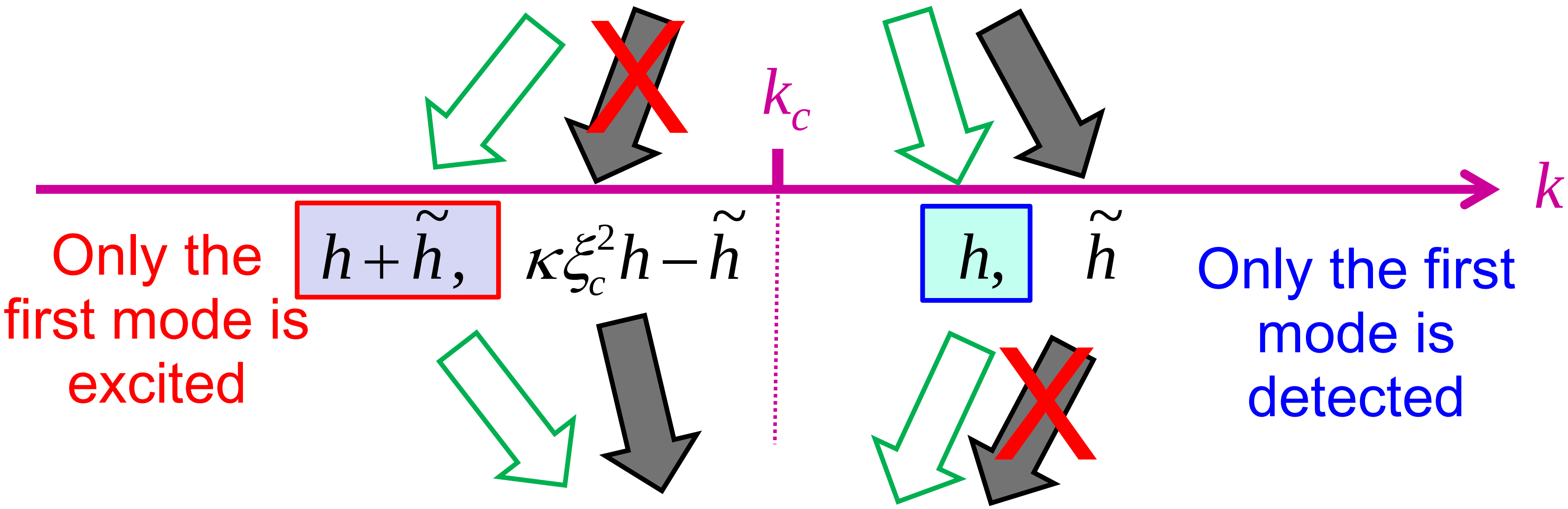
$C \neq 1$ is important.

Eigenmodes are

$$h, \quad \tilde{h}$$

modified dispersion relation due to different light cone

At the GW generation, both h and $\tilde{h}$ are equally excited.



We can detect only h.

Only modes with $k \sim k_c$ picks up the non-trivial dispersion relation of the second mode.

Interference between two modes. ⇒ Graviton oscillations

If the effect appears ubiquitously, such models would be already ruled out by other observations.

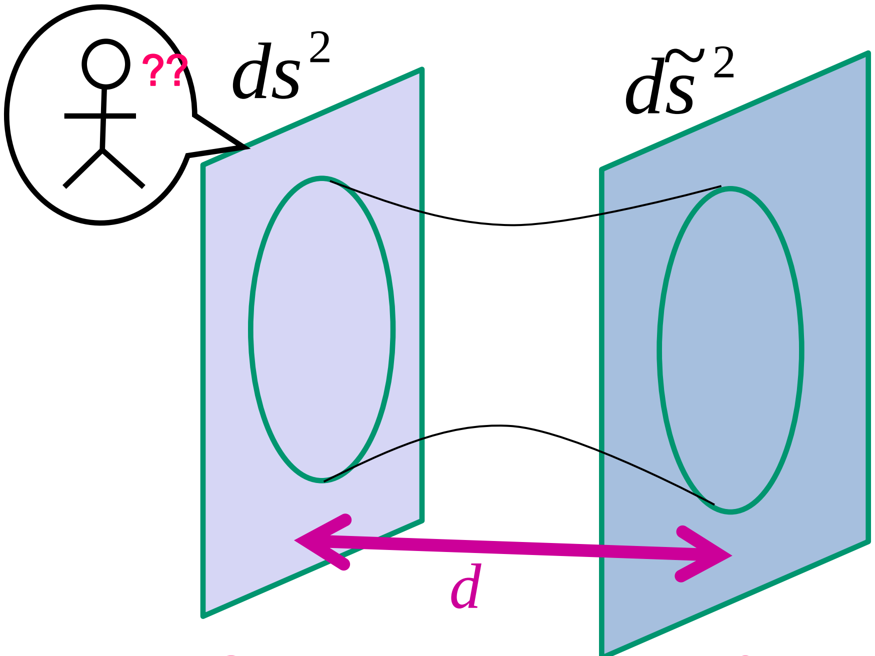
Summary

Gravitational wave observations open up a new window for modified gravity.

Even the radical idea of graviton oscillations is not immediately denied. We may find something similar to the case of solar neutrino experiment in near future.

Although space GW antenna is advantageous for the gravity test in many respects, more that can be tested by KAGRA will be remaining to be uncovered.

Why do we have this attractor behavior, $c \to 1$ and $\xi \to \xi_c$, at low energies?



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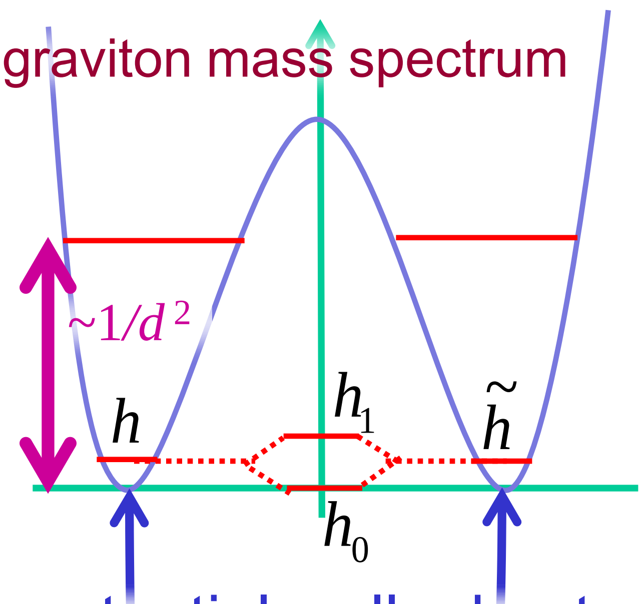


DGP 2-brane model?!

$$+\int d^4x\sqrt{-g}R \;\&\; \int d^4x\sqrt{-\tilde{g}}\tilde{R}$$

$d \to 0$ ⇒ Only first two modes remain at low energy

⇒ $d\tilde{s}^2 = \xi_c^2 ds^2$ ⇒ identical light cone $c = 1$

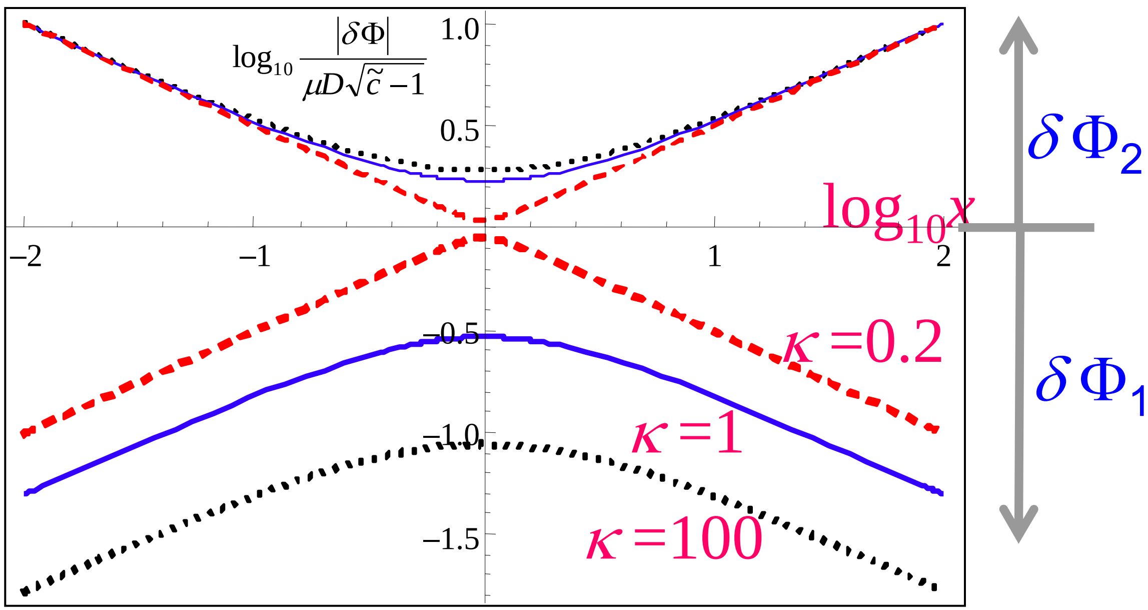
Gravitational wave propagation over a long distance D

Phase shift due to the modified dispersion relation:

$$\delta\Phi_{1,2} \equiv -\frac{\Delta k^2}{2\omega} D = -\frac{\mu D\sqrt{c-1}}{2\sqrt{2x}}\left(1+x \mp \sqrt{1+2x\frac{1-\kappa\xi^2}{1+\kappa\xi^2}+x^2}\right)$$

$$\mu D\sqrt{c-1} \approx HD\sqrt{3\left(1+\kappa\xi_c^2\right)\Omega_0}$$

becomes O(1) after propagation over the horizon distance



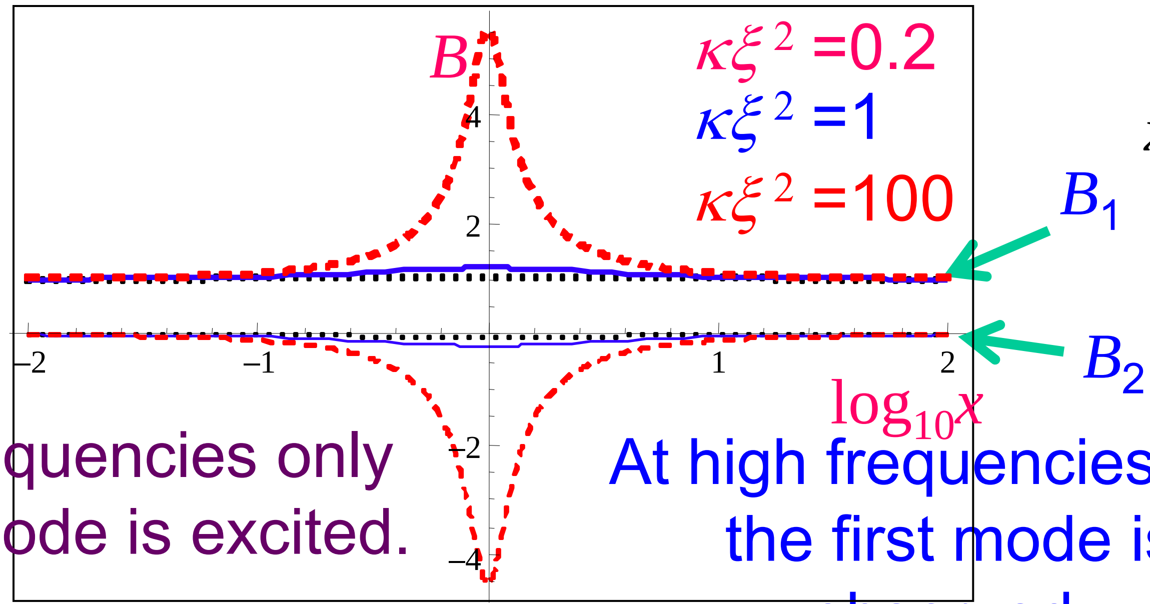
$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

Gravitational wave oscillations

1) At the time of generation of GWs from coalescing binaries, both h and $\tilde{h}$ are equally excited.

2) When we detect GWs, we sense h only.

$$h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$

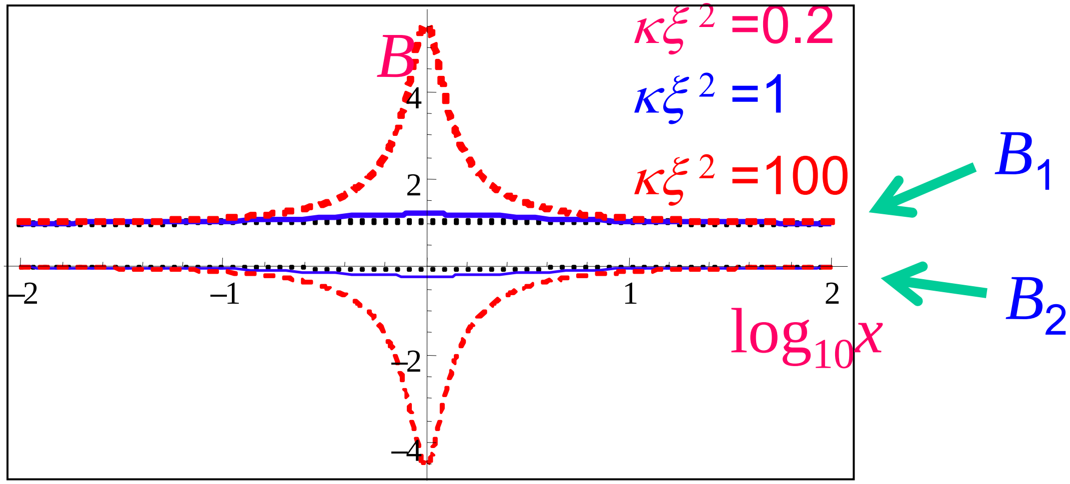


$$x \equiv \frac{2\omega^2(c-1)}{\mu^2}$$

At low frequencies only the first mode is excited.

At high frequencies only the first mode is observed.

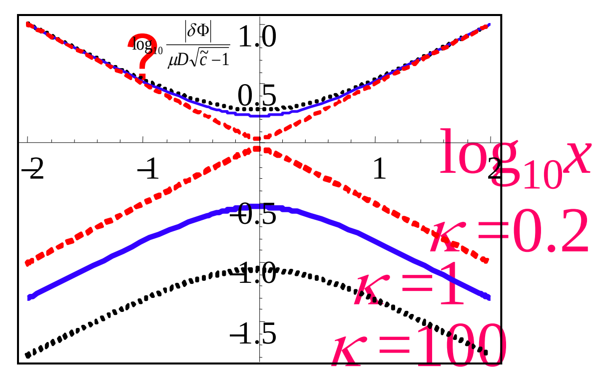
$$h(f) \propto A(f)\left[B_1(f)e^{i\Phi_{GR}(f)+i\delta\Phi_1(f)} + B_2(f)e^{i\Phi_{GR}(f)+i\delta\Phi_2(f)}\right]$$



Graviton oscillations occur only around the frequency

$$\omega_{GW} \approx \frac{\mu^2}{\sqrt{6(1+\kappa\xi_c^2)\Omega_0}\,H} = 100\text{Hz}\left(\frac{1+\kappa\xi_c^2}{100}\right)^{-1/2}\left(\frac{\mu}{(0.08\,pc)^{-1}}\right)^4 \quad \Leftarrow \quad x \approx 1$$

Phase shift is as small as $HD\sqrt{3(1+\kappa\xi_c^2)\Omega_0}$



No, $x << 1$ when the GWs are propagating the inter-galactic low density region.

Summary

Gravitational wave observations give us a new probe to modified gravity.

Even graviton oscillations are not immediately denied, and hence we may find something similar to the case of solar neutrino experiment in near future.

Although space GW antenna is advantageous for the gravity test in many respects, we should be able to find more that can be tested by KAGRA.