Wormhole solutions in KGB framework

Rotondo Marcello（名古屋大学大学院　理学研究科）

Abstract

In this work we will look for wormhole solutions to the Einstein field equations in the contest of the Kinetic Gravity Braiding model. This model allows the introduction of interactions up to the second derivative order for the scalar field that do not lead to higher orders in the equation of motion, opening to a wide range of new dynamical features. We will consider a particular subclass of KGB models, i.e. the conformal galileon, giving one example of its capability to violate the null energy condition as required for wormholes. We will numerically solve Einstein field equations in this framework and show some preliminary results.

1 Introduction

Wormholes represent a class of solutions to Einstein field equations (EFE) that refer to a topological feature of the space-time in which distant regions of the same universe (or different universes) are connected. The metric for a wormhole in its static, spherically symmetric form can be expressed as

$$ds^2 = -e^{-2\Phi(r)}dt^2 + dr^2 + R(r)^2 d\Omega^2 \qquad (1)$$

with $\Phi(r)$ and $R(r)$ generic functions of the radius r referred to as "redshift function" and "shape function", respectively.

It can be shown that the wormhole solution requires a stable violation of the *null energy condition* (NEC)

$$T_{\mu\nu}k^\mu k^\nu \geq 0 \qquad (2)$$

where k^μ and k^ν are any two future-oriented timelike vectors.

Usually the stable violation of NEC brings to ghost degrees of freedom, gradient instabilities (imaginary speed of sound), etc. There is nevertheless a class of theories that allow this violation. In this work, we consider one of its simplest and yet large subclasses, a minimally coupled set of models that doesn't require any direct coupling with the Riemann tensor: the Kinetic Gravity Braiding (KGB) models.

2 Kinetic Gravity Braiding

The kinetic braiding constitutes a large class of scalar-tensor models with interactions containing second derivatives of the scalar field but not leading to additional degrees of freedom in the equation of motion. For the sake of simplicity, from now on we will consider a radial scalar field $\phi(r)$ and the Lagrangian in the form

$$L = K(\phi, X) + G(\phi, X)\nabla^\mu\nabla_\mu\phi \qquad (3)$$

where $X = \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $K(\phi, X)$, $G(\phi, X)$ are generic functions.

The term kinetic braiding comes from the presence in the Lagrangian of the term $G(\phi, X)\nabla^\mu\nabla_\mu\phi$ where, due to the presence of the Cristoffel symbol in the covariant d'Alembertian operator, there is a coupling between the derivative of the metric and the derivative of the field.

An other form for the Lagrangian is obtained through integration by parts of the scalar field contribution to the action and leads to

$$P = K - \left[\left(\nabla^\lambda \phi \right) \nabla_\lambda \right] G = K - 2X * G_\phi - G_X \nabla^\lambda \phi \nabla_\lambda X \quad (4)$$

where the presence of the $G_X \neq 0$ term is the reason why this model deviates from the k-essence model and displays peculiar features. From the physical point of view, this is reflected in a deviation from the perfect fluid behavior.

The equation of motion for the scalar field is

$$P_\phi - \nabla_\mu \left(\left(L_X - G_\phi \right) \nabla^\mu \phi - \nabla^\mu G \right) = 0 \quad (5)$$

The energy momentum tensor for the scalar field is

$$T_{\mu\nu} = L_X \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} P - \nabla_\mu G \nabla_\nu \phi - \nabla_\nu G \nabla_\mu \phi \quad (6)$$

We observe that both the equation of motion and the EMT contain second derivatives of the field and the metric, so that the system is not diagonal in the second order. This kinetic braiding is therefore essential and cannot be diagonalized through a conformal transformation.

2.1 Conformal galileon NEC violation

An example of NEC violation allowed by the kinetic mixing between the scalar field and the metric tensor in the KGB framework is given by the bouncing model for a spatially flat Friedmann universe. The study of this kind of model may be useful in order to understand the current accelerated expansion of the universe as well as the inflation mechanism. One of these models is the *conformal galileon*, which is a KGB model with the arbitrary functions $G(\phi, X)$ and $K(\phi, X)$ set to

$$K(\phi, X) = -2f^2 e^{2\phi} X + \frac{2f^3}{\Lambda^3} X^2 \quad (7)$$

$$G(\phi, X) = \frac{2f^3}{\Lambda^3} X \nabla^\mu \nabla_\mu \phi \quad (8)$$

where Λ and f are constants with mass dimension one. It can be shown [3] that all the evolution trajectories for this model that have bounced in the past, passing from a phase of contraction to a phase of expansion, will also eventually approach and cross the boundary where the speed of sound vanishes $c^2 = 0$ and becomes imaginary, leading to NEC violation.
There is the possibility that this bouncing solution may in future studies be related with wormhole dynamics as well. The Schwarzschild solution to EFE is the first and simplest wormhole solution [4]. It represents a non-traversable wormhole, since the size of its throat is not stable: as time passes it expands from zero throat circumference to a maximum circumference radius where it bounces and contracts again to zero circumference. There is the possibility that an inverted G-Bounce solution may be applied to this kind of dynamics.

3 Results

Einstein field equations for a static and spherically symmetric wormhole solution with a metric of the form (1) in the case of a scalar field obeying to the galileon KGB lagrangian introduced above constitute a system of three second order differential equations in three variables ($R(r), \Phi(r)$ and $\phi(r)$), to which the equation of motion is to be added. It is possible to show that the differentiation of the pure radial Einstein field equation is automatically satisfied by the other equations, so it can be taken as a constraint equation. Furthermore, the redshift function $\Phi(r)$ appears only on the first and second order of derivation. With the substitution $\Phi'(r) \to u(r)$, the system reduces to two second order differential equations in $R(r)$ and $\phi(r)$, one in the first order of $u(r)$ and one constraint equation.

Referring to the two regions connected by the wormhole as '-' and '+', the boundary conditions at r=0 are simply given by:

$$R_{\pm}(0) = R_0 \tag{9}$$

$$R'_{\pm}(0) = 0 \tag{10}$$

$$u_{\pm}(0) = 0 \tag{11}$$

$$\phi_{\pm}(0) = \phi_0 \tag{12}$$

$$\phi'_{\pm}(0) = \pm\phi'_0 \tag{13}$$

where the value ϕ'_0 is fixed solving the constraint equation at r=0. Preliminary results are represented in the attached graphs.

4 Discussion

The metric is Minkovski at infinity, as it is expected to be. The shape function $R(r)$ doesn't vanish as the origin of the wormhole is approached, leading to a finite-size throat and therefore possibly to a traversable wormhole. The solution for the $\Phi(r)$ function is asymmetric, showing that the features of wormholes (in particular the "flow of time" which the redshift function accounts for) may actually be different [4] depending on which region the observer is placed in. Last, the scalar field vanishes at infinity and is non-vanishing inside the wormhole. It doesn't show symmetry for inversion of the radial coordinate, leading to different energy distributions on the two external regions.

5 Conclusion

The results obtained, though preliminary, are coherent with a wormhole solution. Our future work will be directed to the quest for more general solutions for the KGB model with looser restrictions on the form of the Lagrangian than those assumed in this work. Furthermore, the possible application of the G-bounce model to non-traversable wormholes shall be investigated.

6 Figures

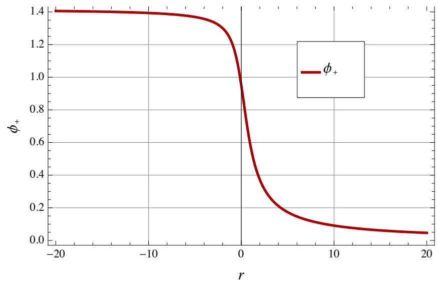

図 1: $\phi(r)$

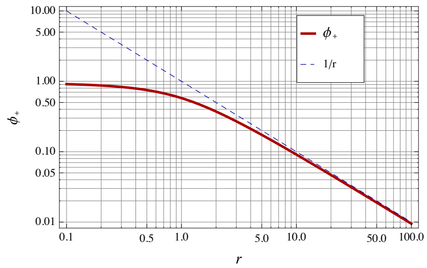

図 2: $\phi_+(r)$

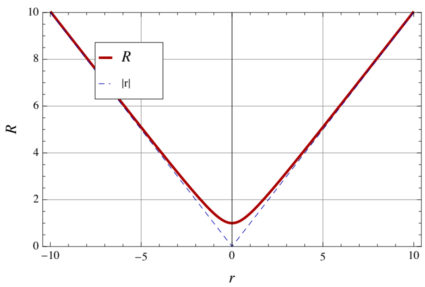

図 3: $u_+(r)$

Acknowledgement

I wish to thank Dr.Yoo Chulmoon (Nagoya Univ.) and Prof.Tsutomu Kobayashi (Tokyo Univ.) for

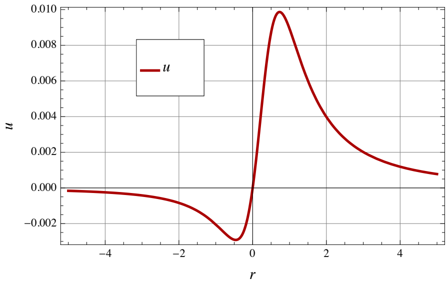

図 4: $u(r)$

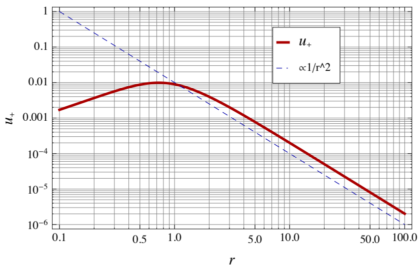

図 5: $u_+(r)$

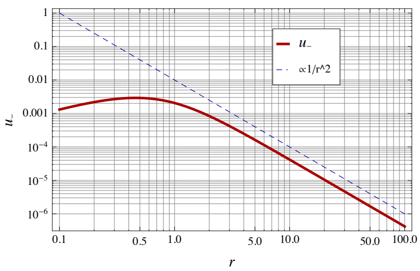

図 6: $u_-(r)$

their guidance and collaboration.

Reference

C.Deffayet, O.Pujolas, I.Sawicki 2010. JCAP 1010:026

M.S.Morris, K.S.Thorne 1988. Am.J.Phys 56(5)

D.A.Easson,I.Sawicki and A.Vikman 2011. JCAP11(2011)021

M.Visser 1996. AIP Press, American Institute of Physics