最も一般的なスカラー・テンソル理論における FRW背景宇宙でのVainshtein機構

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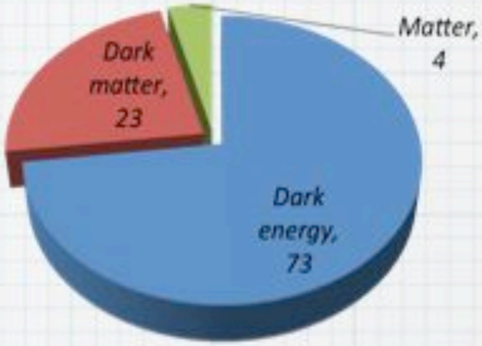
Accelerating universe

- The present cosmological observations indicate the accelerated expansion of the universe
  - ☑ Type Ia supernovae
  - ☑ Cosmic microwave background
  - ☑ Large scale structures



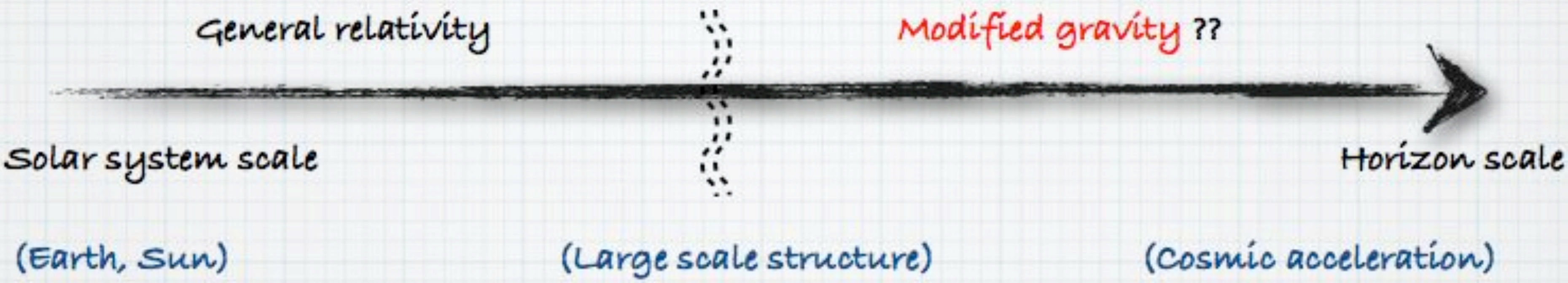
- Possible candidates
  - ☑ Cosmological constant $G_{\mu\nu}+\Lambda g_{\mu\nu}=8\pi G T_{\mu\nu}$
  - ☑ Dark energy $G_{\mu\nu}=8\pi G\,T_{\mu\nu}+8\pi G\,T_{\mu\nu}^{\rm DE}$
  - ☑ Modification of gravity

$$S=\frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\Big[R+f(R)\Big]$$

$$S=\frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left[R-\frac{\omega}{\phi}(\nabla\phi)^2-V(\phi)\right]$$

etc...

Alternative : Modification of gravity



Modified gravity must recover "general relativity behavior" at short distances

→ Screening mechanism

Example: Vainshtein mechanism
Chameleon mechanism
Symmetron mechanism

Vainshtein Mechanism

- ✓ **Galileon example** (Deffayet et al. '10)

self-accelerating solution $r_c \sim \mathcal{O}(H_0^{-1})$

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{r_c^2}{2M_{\rm Pl}}(\partial\phi)^2\Box\phi + \mathcal{L}_m[\psi, g_{\mu\nu}]$$

- ✓ Scalar field profile

Vainshtein radius

$$r_V \sim (r_s r_c^2)^{1/3}$$

Solar system scale → Horizon scale

"Nonlinear"

$$F_\phi \sim \frac{GM}{r^2}\left(\frac{r}{r_V}\right)^{3/2} \ll \frac{GM}{r^2}$$

ϕ is effectively weakly coupled to matter

"Linear"

$$F_\phi \sim \frac{GM}{r^2}$$

ϕ is strongly coupled to matter

Inside the Vainshtein radius, GR can be recovered through Vainshtein mechanism

The most general scalar-tensor theory

✓ Horndeski found the most general Lagrangian whose EOM is second-order differential equation for ϕ and $g_{\mu\nu}$ (also known as Generalized galileon)

Deffayet, Gao, Steer (2011)
Kobayashi, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011)
Horndeski, Int. J. Theor. Phys. 10,363 (1974)

$$\mathcal{L}_2 = K(\phi, X)$$

→ K-essence term $\mathcal{L}_2 \supset (\partial\phi)^2,\ V(\phi)$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

→ Cubic galileon term $\mathcal{L}_3 \supset (\partial\phi)^2\Box\phi$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)]$$

→ Einstein-Hilbert term $\mathcal{L}_4 \supset (M_{\rm Pl}^2/2)R$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\Big[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\Big]$$

→ Non-minimal derivative coupling $\mathcal{L}_5 \supset G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$

(Germani et al. 2011; Gubitosi, Linder 2011)

$$X = -(\partial\phi)^2/2, \quad G_{iX} = \partial G_i/\partial X$$

QUESTION :

Does Vainshtein mechanism work
in the most general second-order
scalar-tensor theory
in a cosmological background???

Formulation

✓ Newtonian gauge and perturbations

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)d\mathbf{x}^2$$

$$\phi \;\to\; \phi(t) + \delta\phi(t,\mathbf{x}), \qquad Q \equiv (H/\dot{\phi})\delta\phi$$

$$\rho_{\rm m} \;\to\; \rho_{\rm m}(t)[1+\delta(t,\mathbf{x})] \qquad \epsilon = \Psi, \Phi, \text{ and } Q \ll 1$$

✓ In field equations,

Neglect

Quasi-static approximation

$$\partial_t \ll \partial_x$$

$$\text{EOM} \supset \left\{ \text{"}\cancel{\textit{mass terms}}\text{"}, \text{"}\cancel{\textit{time derivative terms}}\text{"}, \right.$$

$$\left. \boxed{\left(L(t)^2\,\partial^2\epsilon\right)^n}, \cancel{\left(L(t)\,\partial\epsilon\right)^m} \;\ldots \right\}$$

$L(t) \sim \mathcal{O}(H^{-1})$

higher-order terms

→ Picking up the terms like $\partial^2\epsilon,\ (\partial^2\epsilon)^2,\ (\partial^2\epsilon)^3,\ (\partial^2\epsilon)^4,\ \delta$

Field equations

✓ Traceless part of the Einstein equations

$$\nabla^2\left(\mathcal{F}_T\Psi-\mathcal{G}_T\Phi-A_1Q\right)$$
$$=\frac{B_1}{2a^2H^2}\mathcal{Q}^{(2)}+\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2Q-\partial_i\partial_j\Phi\partial^i\partial^jQ\right)$$

✓ 00 component of the Einstein equation

$$\mathcal{G}_T\nabla^2\Psi=\frac{a^2}{2}\rho_{\rm m}\delta-A_2\nabla^2Q$$
$$-\frac{B_2}{2a^2H^2}\mathcal{Q}^{(2)}-\frac{B_3}{a^2H^2}\left(\nabla^2\Psi\nabla^2Q-\partial_i\partial_j\Psi\partial_i\partial_jQ\right)-\frac{C_1}{3a^4H^4}\mathcal{Q}^{(3)}$$

✓ Scalar field equation

$$A_0\nabla^2Q-A_1\nabla^2\Psi-A_2\nabla^2\Phi+\frac{B_0}{a^2H^2}\mathcal{Q}^{(2)}-\frac{B_1}{a^2H^2}\left(\nabla^2\Psi\nabla^2Q-\partial_i\partial_j\Psi\partial^i\partial^jQ\right)$$
$$-\frac{B_2}{a^2H^2}\left(\nabla^2\Phi\nabla^2Q-\partial_i\partial_j\Phi\partial^i\partial^jQ\right)-\frac{B_3}{a^2H^2}\left(\nabla^2\Phi\nabla^2\Psi-\partial_i\partial_j\Phi\partial^i\partial^j\Psi\right)$$
$$-\frac{C_0}{a^4H^4}\mathcal{Q}^{(3)}-\frac{C_1}{a^4H^4}\mathcal{U}^{(3)}=0$$

A_i, B_i, and C_i are functions of K, G_3, G_4, G_5

$$\mathcal{Q}^{(2)}\equiv\left(\nabla^2Q\right)^2-\left(\partial_i\partial_jQ\right)^2$$
$$\mathcal{Q}^{(3)}\equiv\left(\nabla^2Q\right)^3-3\nabla^2Q\left(\partial_i\partial_jQ\right)^2+2\left(\partial_i\partial_jQ\right)^3$$
$$\mathcal{U}^{(3)}\equiv\mathcal{Q}^{(2)}\nabla^2\Phi-2\nabla^2Q\partial_i\partial_jQ\partial^i\partial^j\Phi+2\partial_i\partial_jQ\partial^j\partial^kQ\partial_k\partial^i\Phi$$

Spherically Symmetric Case

✓ EOM for gravity and scalar field can be integrated once,

$$c_h^2\frac{\Psi'}{r}-\frac{\Phi'}{r}-\alpha_1\frac{Q'}{r}=\frac{\beta_1}{H^2}\left(\frac{Q'}{r}\right)^2+2\frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{Q'}{r}$$

$$\frac{\Psi'}{r}+\alpha_2\frac{Q'}{r}=\frac{1}{8\pi\mathcal{G}_T}\frac{\delta M(t,r)}{r^3}-\frac{\beta_2}{H^2}\left(\frac{Q'}{r}\right)^2-2\frac{\beta_3}{H^2}\frac{\Psi'}{r}\frac{Q'}{r}-\frac{2}{3}\frac{\gamma_1}{H^4}\left(\frac{Q'}{r}\right)^3$$

$$\alpha_0\frac{Q'}{r}-\alpha_1\frac{\Psi'}{r}-\alpha_2\frac{\Phi'}{r}=2\Bigg[-\frac{\beta_0}{H^2}\left(\frac{Q'}{r}\right)^2+\frac{\beta_1}{H^2}\frac{\Psi'}{r}\frac{Q'}{r}+\frac{\beta_2}{H^2}\frac{\Phi'}{r}\frac{Q'}{r}+\frac{\beta_3}{H^2}\frac{\Phi'}{r}\frac{\Psi'}{r}+\frac{\gamma_0}{H^4}\left(\frac{Q'}{r}\right)^3+\frac{\gamma_1}{H^4}\frac{\Phi'}{r}\left(\frac{Q'}{r}\right)^2\Bigg]$$

α_i, β_i, and γ_i are functions of K, G_3, G_4, G_5

where

$$c_h^2\equiv\mathcal{F}_T/\mathcal{G}_T$$

(Propagation speed of the gravitational waves)

Case 1 :	$G_{4X}=0,\ G_5=0$	$a\,Q'^2+b\,Q'+c=0$
Case 2 :	$G_{5X}=0$	$a\,Q'^3+b\,Q'^2+c\,Q'+d=0$
Case 3 :	$G_{5X}\neq 0$	$a\,Q'^6+b\,Q'^5+c\,Q'^4+d\,Q'^3+e\,Q'^2+f\,Q'+g=0$

Linear Solution

✓ For sufficiently large r, (linearlize ψ, Φ, and φ)

(time-dependent)
effective gravitational coupling $G_{\rm eff}^{\rm (Linear)} (\neq G_N)$

$$\Phi' = \frac{1}{8\pi\mathcal{G}_T}\frac{c_h^2\alpha_0 - \alpha_1^2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\delta M}{r^2}$$

$$\Psi' = \frac{1}{8\pi\mathcal{G}_T}\frac{\alpha_0 + \alpha_1\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\delta M}{r^2}$$

$$Q' = \frac{1}{8\pi\mathcal{G}_T}\frac{\alpha_1 + c_h^2\alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2\alpha_2)\alpha_2}\frac{\delta M}{r^2}$$

In general, the gravitational coupling $G_{\rm eff}$ in the Poisson equation is different from Newton's constant on cosmological scales

Case 1 : $G_{4X} = 0,\ G_5 = 0$

Propagation speed of the gravitational waves

$$c_h^2 = 1$$

✓ Kinetic gravity braiding with non-minimal coupling (Deffayet et al. 2010)

$$\mathcal{L} = G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\Box\phi$$

✓ Scalar field solution

$$\frac{Q'}{r} = \frac{H^2}{\mathcal{B}}\left(\sqrt{1+\left(\frac{r_V}{r}\right)^3} - 1\right)$$

Vainshtein radius

$$r_V = \left(\frac{2\mathcal{B}\mathcal{C}\mu}{H^2}\right)^{1/3}$$

r_V~100 pc for sun

✓ Inside the Vainshtein radius r_V

$$Q' \simeq \frac{H}{\mathcal{B}}\sqrt{\frac{2\mathcal{B}\mathcal{C}\mu}{r}} \ll \frac{GM}{r^2}$$

Gravitational potentials

$$\Psi' = \Phi' \simeq \frac{1}{16\pi G_4(t)}\frac{\delta M}{r^2}$$

Newton's constant $G_N(t)$

PPN parameter $\gamma \equiv \Psi'/\Phi' = 1$

Friedmann equation

$$3H^2 = 8\pi G_{\cos}\left(\rho_m + \rho_\phi\right)$$

$$G_{\cos} = G_N$$

Time-dependence of G_N can be tested by BBN

$$\left|1 - \frac{G_N|_{\rm BBN}}{G_N|_{\rm now}}\right| \lesssim 0.1$$

(Uzan 2011)

Case 2 : $G_{5X} = 0$

✓ Lagrangian

$$\mathcal{L} = K(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right] + G_5(\phi)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi$$

does not depend on the kinetic term X

Propagation speed of the gravitational waves
$c_h^2 = 1 + 2\beta_1 \neq 1$

✓ Scalar field equations

$$(Q')^3 + \mathcal{C}_2 H^2 r (Q')^2 + \left(\frac{\mathcal{C}_1}{2}H^4 r^2 - H^2 \mathcal{C}_\beta \frac{\mu}{r}\right) Q' - \frac{H^4 \mathcal{C}_\alpha \mu}{2} = 0$$

C_1, C_2, C_α, and C_β are functions of K, G_3, G_4, G_5

$$\mu \equiv \frac{\delta M}{8\pi \mathcal{G}_T}$$

✓ 3 possible solutions at short distance can be matched to the linear solution

$$Q' \simeq +H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}, \quad -H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}, \quad -\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta}\frac{H^2 r}{2}$$

Case 2 : $G_{5X} = 0$

✓ Solution I $\quad Q' \simeq \pm H\sqrt{\mathcal{C}_\beta \frac{\mu}{r}}$

Gravitational potentials

$$\Psi' = \Phi' \simeq \frac{C_\Psi(t)}{8\pi\mathcal{G}_T(t)}\frac{\delta M}{r^2}$$

Newton's constant $G_N(t)$

PPN parameter $\quad \gamma \equiv \Psi'/\Phi' = 1$

Friedmann equation

$$3H^2 = 8\pi G_{\cos}(\rho_m + \rho_\phi)$$

$$G_{\cos} = G_N$$

Time-dependence of G_N can be tested by BBN

✓ Solution II $\quad Q' \simeq -\frac{\mathcal{C}_\alpha}{\mathcal{C}_\beta}\frac{H^2 r}{2}$

Gravitational potentials

$$\Phi' \simeq c_h^2 \Psi' \simeq \frac{c_h^2}{8\pi\mathcal{G}_T}\frac{\delta M}{r^2}$$

Newton's constant $G_N(t)$

Solar-system tests (Will 2005)

$$|1-\gamma| < 2.3 \times 10^{-5}$$

PPN parameter

$$\gamma \equiv \frac{\Psi'}{\Phi'} = \frac{1}{c_h^2} (\neq 1)$$

propagation speed of gravitational waves

Case 3 : $G_{5X} \neq 0$

✓ At sufficiently small scales, the inverse square law for the gravitational force

$$\Psi'(r),\ \Phi'(r)\ \propto\ \frac{1}{r^2}$$

is not a solution anymore !

→ The coupling between the Einstein tensor and the kinetic term $XG_{\mu\nu}$ leads to a strong modification of gravity even at short distances

The Vainshtein mechanism no longer works in the presence of G_{5X} !!
(Observationally ruled out...)

Summary

- The most general second-order scalar-tensor theory
  - ☑ Second-order differential equation
  - ☑ Self-accelerating solution
  - ☑ No ghost-instability

- Vainshtein screening successfully operates in the most general second-order scalar-tensor theory, but
  - ☑ Newton's constant G=G(t)
  - ☑ constrained from PPN and BBN
  - ☑ inverse-square law can not be reproduced at small scales if $G_{5X} \neq 0$